Math 103. Section 205. Quiz 2a
March 1st, 2007

NAME:

STUDENT NUMBER:

Calculators and other electronic devices are neither allowed nor required for this test. This exam has 2 pages.

Compute the following integrals:

1. \[ \int \frac{dx}{x^2 + x - 2} \]

Solution: \[ \int \frac{dx}{x^2 + x - 2} = \int \frac{dx}{(x - 1)(x + 2)}. \]

We will solve this problem using partial fractions

\[ \frac{1}{(x - 1)(x + 2)} = \frac{A}{x - 1} + \frac{B}{x + 2} = \frac{A(x + 2) + B(x - 1)}{(x - 1)(x + 2)} \]

\[ \begin{cases} A + B = 0 \\ 2A - B = 1 \end{cases} \]

From first equation \( A = -B \), adding both equations, \( 3A = 1 \).

Therefore, \( A = \frac{1}{3} \) and \( B = -\frac{1}{3} \).

(We can also solve it by plugging \( x = -2 \) and \( x = 1 \) in \( A(x + 2) + B(x - 1) = 1 \).)

\[ \int \frac{dx}{x^2 + x - 2} = \int \left( \frac{1}{3(x - 1)} - \frac{1}{3(x + 2)} \right) dx = \frac{1}{3} \ln |x - 1| - \frac{1}{3} \ln |x + 2| + C \]

2. \[ \int x \sin(x) dx \]

Solution: We solve this problem using integration by parts. Let \( u = x \) and \( dv = \sin(x) \). Then \( du = dx \) and a possible \( v = -\cos(x) \).

\[ \int x \sin(x) dx = x(-\cos(x)) - \int (-\cos(x)) dx = -x \cos(x) + \int \cos(x) dx = -x \cos(x) + \sin(x) + C \]
3. \( \int \frac{\ln(3x)}{x} dx \)

**Solution:** We solve this problem by using substitution \( u = \ln(3x) \). Then \( du = \frac{dx}{x} \).

\[
\int \frac{\ln(3x)}{x} dx = \int u du = \frac{u^2}{2} + C = \frac{\ln^2(3x)}{2} + C
\]

4. \( \int_0^2 \sqrt{4 - x^2} dx \) (hint: \( x = 2 \sin u \), \( \int_0^{\pi/2} \cos^2 y dy = \frac{\pi}{4} \))

**Solution:** We do the substitution \( x = 2 \sin u \). Then \( dx = 2 \cos u du \)

\[
\int_0^2 \sqrt{4 - x^2} dx = \int_{\sin^{-1}(0)}^{\sin^{-1}(2)} \sqrt{4 - 4 \sin^2 u} \cdot 2 \cos u du = \int_0^{\pi/2} 4 \cos^2 u du = \frac{4\pi}{4} = \pi
\]

The problem can be also solved by observing that we are computing one fourth of the area of the circle of radius 2.