Variational approach to fluid-structure interactions: friction, constraints, Darcy’s law and poromechanics.

This work is motivated by the study of mechanics of elastic porous media filled with fluid. Using a variational geometric approach coupled with the Lagrange-d’Alembert’s method for friction forces, we derive the equations of motion for both the elastic media and the fluid inside the media. We then study some simplified cases such as a pendulum with a moving viscous droplet. We show that the analogue of Darcy’s law in these simplified models comes from the short-term convergence to a ‘constraint manifold’ in a singular perturbation problem, and the following long-term dynamics on that manifold. The resulting Darcy’s law can reduce to either holonomic or non-holonomic constraint for the motion, depending the physical realization. We also demonstrate that care must be taken in formulating Darcy’s law as the long-term dynamics can change drastically for small perturbations of the system. We discuss the relevance of these results for poromechanics and consider some simplified physical cases of the porous media motion.

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