Math 422 Winter 2008 - Sample Questions

Problem 1. Let C be a linear code over $\mathbb{Z}_7$ with parity check matrix

$$H = \begin{bmatrix} 2 & 0 & 1 & 2 & 1 \\ 3 & 2 & 0 & 1 & 4 \\ 4 & 2 & 1 & 3 & 6 \end{bmatrix}$$

a) Find the standard form S of H.

b) Find a generator matrix for the code D with parity check matrix S. Is $D = C$?

Problem 2. Let C be the following 3-ary linear code over $\mathbb{Z}_3$: $C = \{\mathbf{0}, 121, 212\}$.

a) Find a basis for the dual code $C^\perp$. Which of the following statements is true:

$$C = C^\perp, C \subseteq C^\perp, C^\perp \subseteq C$$

Justify your answer.

b) Find a parity check matrix H for C.

c) Write down a syndrome table for C, that is, for each coset of C pick a minimum weight element and compute its syndrome using a parity check matrix for C; list the result.

d) Using your table from c), decode the received vector 222.

e) Is C a perfect code?

f) Is C, as a linear code, equivalent to the ternary repetition code of length 3?

Problem 3. Recall the Plotkin bound: A binary (n, M, d)-code where d is even and $2d > n$ satisfies $M \leq 2[d/(2d - n)]$. Use this to prove that $A_2(2k + 1, 2k) = 2$ (for $k \geq 2$). Conclude that $A_2(n, n - 1) = 2$ whenever $n \geq 4$ (i.e. also in the case where $n - 1$ is odd). (**Hint:** Do not forget to explain why there is always a $(n, 2, n - 1)$-code for $n \geq 4$!)

Problem 4. Prove that if C is a q-ary (n, M, d)-code where $d > 0$ is even, then C is not perfect. That is, there is no $t > 0$ such that F_q^n is the disjoint union of spheres of radius t around the codewords.
(**Hint:** Construct a vector $\mathbf{y}$ that is not contained in a *unique* sphere of radius t around a codeword.)

Problem 5. For each of the following statements, indicate whether it is true or false. (No justification required)

a) Two linear codes are equivalent if they are contained in the same F_q^n and have the same dimension.

b) A generator matrix of a linear $[n, k]$-code has rank $n - k$.

c) A generator matrix cannot contain a column of all zeros.

d) The sphere $S(\mathbf{0}, n)$ with radius n around $\mathbf{0}$ is equal to all of F_q^n.

e) There is no linear binary $(12, 36, 6)$-code.

Problem 6. Suppose you are into Hockey games and would like to place a bet on the outcome of 13 Hockey games. The games are numbered 1 through 13. Mathematically, a bet consists of a sequence of 13 symbols out of $0, 1, 2$, where 0 means draw, 1 means the home team wins, 2 means the away team wins. Can you describe a betting strategy to guarantee a second place? To be precise, can you find a number M and a strategy for placing M bets such that the final outcome differs at most in one game from one of your bets? Is your strategy optimal, i.e. is M minimal?
(**Hint:** Hamming code?)