PIMS Algebra Summer School 2007 - Abstracts

Jason Bell (Simon Fraser University) : *GK dimension and birational equivalence of algebras*
GK dimension is a noncommutative analogue of Krull dimension, which has been an instrumental invariant in the study of noncommutative algebras. In this talk, we give the basic theory of GK dimension and the work that has been done on algebras of low GK dimension. We specifically consider domains over an algebraically closed field. In the commutative case, an integral domain has a field of quotients; similarly, a noncommutative domain of finite GK dimension has a division algebra of quotients. In analogy with the commutative case, one can therefore try to give a "birational classification of noncommutative surfaces" (i.e., classify the quotient division algebras of finitely generated domains of GK dimension 2 up to isomorphism). We discuss progress on this problem.

Anders Buch (Rutgers University): *Quantum K-theory of Grassmannians*
Abstract: The Gromov-Witten invariants of a homogeneous space X give the number of rational curves of fixed degree that meet three general Schubert varieties, at least when this number is finite. When there are infinitely many such curves, then the moduli space of (stable) parametrizations of the curves is a projective variety. The K-theoretic Gromov-Witten invariants are the Euler characteristic of such varieties, and were used by Y.-P. Lee and Givental to define a quantum K-theory ring of X. I will present structure theorems for this ring when X is a Grassmann variety of type A, and a formula for the K-theoretic Gromov-Witten invariants that generalizes earlier work with Kresch and Tamvakis. This is joint work with L. Mihalcea.

Sunil Chebolu (University of Western Ontario): *Some new invariants for group algebras*
Abstract: Let G be a finite p-group and k be a field of characteristic p. Motivated by the generating hypothesis – a deep conjecture in stable homotopy theory, we study maps between finite dimensional kG-modules that are invisible to Tate cohomology — such maps are called ghosts. A natural question is to find the smallest integer l such that the composition of any l ghosts is stably trivial, i.e., factors through a projective. I will talk about these new integer invariants for kG with a pleasant mix of techniques from group theory, representation theory, triangulated category theory and constructions motivated from homotopy theory.
This is joint work with Minac and Christensen.

Vladimir Chernousov (University of Alberta): *Purity of G_2-torsors*
Abstract. Joint work with I. Panin. We describe the image of the mapping $H^1(R, G) \to H^1(K, G)$ where G is a split group of type G_2, R is a regular local ring containing a field of characteristic zero and K is the fraction field of R.

Fernando Cukierman (University of Buenos Aires): *Irreducible components of spaces of algebraic singular foliations*
Abstract: Let $F_q(r, d)$ denote the space of algebraic integrable distributions of codimension q and degree d on a complex projective space of dimension r. We plan to describe the classically known irreducible components of $F_q(r, d)$ for $q = 1$ and some recently found components for $q \geq 1$.

David Harari (University of Paris XI): *Local-global principles for algebraic groups*
Abstract: I will discuss several results and conjectures about Galois cohomology of algebraic groups over local fields and global fields. In particular miscellaneous generalizations of Poitou-Tate exact sequence will be given.

Michael Lau (University of Windsor): *Forms of Conformal Superalgebras*
Abstract: Conformal superalgebras describe symmetries of superconformal field theories and come equipped with an infinite family of products. They also arise as singular parts of the vertex operator superalgebras associated with some well-known Lie structures (e.g. affine, Virasoro, Neveu-Schwarz).
In joint work with Arturo Pianzola, we classify forms of conformal superalgebras using a non-abelian Cech-like cohomology set. As the products in scalar extensions are not given by linear extension of the products in the base ring, the usual descent formalism cannot be applied blindly. As a corollary, we answer a question of Kac and obtain a rigourous proof of the pairwise non-isomorphism of an infinite family of N=4 conformal superalgebras appearing in mathematical physics.

Nicole Lemire (University of Western Ontario): *Linearisation of Multiplicative Group Actions*
Abstract: Let L be a $\mathbf{Z}G$ lattice and k a field on which G acts trivially. G acts multiplicatively on the quotient field $k(L)$ of the group ring $k[L]$. By constrast, G acts linearly on the field $k(V)$ formed by taking the quotient field of the symmetric algebra of the $\mathbf{Q}G$ vector space $V = \mathbf{Q}L$. The $\mathbf{Z}G$ lattice L is called *linearisable* if there is a G-equivariant field isomorphism between $k(L)$ and $k(\mathbf{Q}L)$. Here we examine bounds on the *degree of linearisability*, a measure of the obstruction for a $\mathbf{Z}G$ lattice to be linearisable. We connect these problems to earlier work with Vladimir Popov and Zinovy Reichstein on the classification of the simple algebraic groups which are Cayley and on determining bounds on the Cayley degree of an algebraic group, a measure of the obstruction for an algebraic group to be Cayley.

James D. Lewis (University of Alberta): *Regulator currents on Milnor complexes*
Abstract: Let $X/\mathbf{C}$ be a smooth projective variety. For integers $k,\ m \geq 0$ we consider a cycle group $\mathrm{CH}^k_{\mathrm{M}}(X, m)$ defined in terms of the Zariski cohomology of the sheaf of Milnor K-groups on X, and a corresponding twisted variant $\mathrm{CH}^k_{\mathrm{TM}}(X, m)$. We construct real logarithmic type maps ("real regulators") on $\mathrm{CH}^k_{(\mathrm{TM})}(X, m)$ with values in Hodge cohomology, and investigate their properties.

Olivier Mathieu (University of Lyon): *On polyzeta values*
Abstract: We describe rapidly convergent series for polyzeta values. Surprisingly, the combinatorics used in the formulas can be described very simply in terms of a symmetric space.

Jan Minac (University of Western Ontario): *Who is murdering candidates seeking promotion in Galois Cohomology?*
Abstract: Recently it was reported that all promising indecomposable cohomology classes from the quotients of absolute Galois groups have disappeared. What happened to them? What does their disappearance tell us about the structure of the Sylow p-subgroups of absolute Galois groups? Can one say when and how they have disappeared? Motivated by these puzzling questions, in joint investigations with Benson, Chebolu, Lemire, Schultz, and Swallow, we describe some small quotients of absolute Galois groups and study the behaviour of Galois cohomology as Galois modules over some cyclic extensions of degree p.

Robert Moody (University of Victoria/Alberta): *Almost periodicity*
Abstract: This talk is about almost periodicity: its formal beginnings in the work of Harald Bohr, its natural connection with dynamical systems, and its appearances in the study of point processes and aperiodic crystals. If there is time I will finish with some interesting connections to number theory.

Raman Parimala (Emory University): *Isotropy of quadratic forms over function fields of p-adic curves*
Abstract: It is an open question whether every quadratic form in 9 variables over function fields of curves over p-adic fields has a nontrivial zero. We shall discuss recent progress on this question for nondyadic fields.

Julia Pevtsova (University of Washington): *Varieties associated with modular representations*
Abstract: The theory of support varieties for modules was originated in a groundbreaking work of Quillen who introduced geometry into the study of cohomological properties of a finite group. We take the idea further and define a family of geometric invariants associated to a given modular representation of a finite group (scheme). At the heart of our construction is a notion of a π-point which extends the concept of a "shifted cyclic subgroup" for an elementary abelian p-group to any finite group scheme.
Our constructions lead us naturally to the study of a special class of modules, "modules of constant Jordan type", for which the non-maximal support variety - the first of our geometric invariants - vanishes. The class of modules of constant Jordan type includes endo-trivial modules and enjoys a variety of nice properties: for example, it is closed under taking tensor products or passing to another module in the same component of the Auslander-Reiten quiver. I shall discuss general properties and particular examples of modules of constant Jordan type and finish with some conjectures and open problems.
This is joint work with J. Carlson and E. Friedlander

Zinovy Reichstein (University of British Columbia): *Tschirnhaus transformations revisited*
Abstract: In this expository talk I will revisit the classical topic of simplifying polynomial equations in one variable. I will discuss 19th century theorems of Hermite, Joubert and Klein, recent results in this area, and some open problems.

Louis Rowen (Bar-Ilan University): *Tropical algebras (Joint with Zur Izhakian)*
Abstract: Max-plus algebras have been a source of recent activity, with applications to combinatorics and geometry. Unfortunately their algebraic structure (as semirings) are far from the usual ring-theoretic structure; for example, $a + a = a$ for all elements a, whereas in rings this can only happen for $a = 0$. A few years ago Itzhakian modified this structure, introducing "ghost" elements. Our current work develops a general algebraic structure theory of these algebras, which we call "supertropical algebras." Some sample theorems: If F is a supertropical field, then the polynomial semiring $F[\lambda]$ satisfies a Euclidean algebra, and is a principal ideal domain (where tropical ideals are suitably defined). Every polynomial can be factored into irreducible polynomials. Under a suitable closure assumption, every irreducible polynomial is either linear or quadratic; in several variables, every tropical ideal is generated by linear polynomials. Although unique factorization fails in several variables, there is a natural explanation of when it fails.

David Saltman (Center for Communications Research, Princeton): *Division Algebras over Surfaces*

Abstract: Division algebras are finite dimensional associative algebras where every nonzero element has a multiplicative inverse. The underlying classical question about them is simple - what do they all look like? A wide variety of mathematical tools, some quite abstract, are used to study division algebras. Often the hardest part is to relate concrete division algebras to more abstract objects like elements in a cohomology group. Our goal is to illustrate this confluence through recent work on division algebras over function fields of surfaces. One way to think about this area is that the classical theory, say over number fields, is "one dimensional" and new interesting things happen when one increases the dimension to two. I hope to use about half the time to talk in a basic way about division algebras and the other half to talk about the more specialized theory over surfaces.

Angelika Welte (University of Ottawa) *Graded Lie algebras and central extensions*

Abstract: Interesting classes of Lie algebras often come endowed with a non-trivial grading, e.g., finite dimensional simple Lie algebras, Kac-Moody Lie algebras, extended affine Lie algebras, Heisenberg Lie algebras etc. The universal central extension of a perfect Lie algebra provides more information than the Lie algebra itself, but many results concerning central extensions are only obtained over a field of characteristic zero. In our talk we want to develop a characteristic independent approach to central extensions of certain graded Lie algebras and show that parts of the classical theory are still valid while others fail to be transferable. Fortunately this failure can be "controlled" in important cases, e.g., Lie algebras graded by a reduced root system. But even in the setting of a root graded Lie algebra over the field of complex numbers we are lead to ask questions not only concerning the Lie algebra, but rather the homological properties of other mostly non-associative structures as Jordan pairs and alternative algebras.