Math 421 Winter 2006
Combinatorics
Course Information

Department of Mathematical and Statistical Sciences
University of Alberta

Lecture Q1: M W F 1:00 - 1:50 CAB 229

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(class notes, handouts, solutions, etc. will be available here)

office hours: Tues & Thurs 3:00 - 5:00 in CAB 679, or by appointment

Textbook:

Applied Combinatorics, 2nd Edition, Prentice Hall, New Jersey, 2002.

by Fred S. Roberts and Barry Tesman

Grading Scheme:

Assignments...20%
Midterm Exam...(in class, Wednesday March 1, 2006)......30%
Final Exam...(2:00 p.m. Wednesday April 19, 2006)........50%

The final grades are not curved. The minimum passing grade (D) corresponds to 50%. The grades of C^-, C, C^+ correspond roughly to 63 - 73%, the grades of B^-, B, B^+ correspond roughly to 74 - 88%, and the grades of A^-, A, A^+ correspond roughly to 89 - 100%. These may vary slightly.

Assignments:

There will be 5 assignments given during the term, one approximately every two weeks. They will consist of between 8 and 10 problems and are to be handed in **at the beginning of class on the due date**.

You are encouraged to discuss the problems and solutions with your classmates, however **each student must submit their own version of the solutions**.

The first page of your assignment should contain only your **Name** and **Lecture Section**.

No late assignments will be accepted.

Deferred Examination:

The deferred final examination for this course will be held on Saturday May 13, from 9:00 until 12:00, in Natural Resources Engineering Facility (NRE) 1 001.

Course Objectives:

There are three basic problems in combinatorics. The problems of **existence, enumeration**, and **optimization**: Is there at least one arrangement of a particular kind? How many such arrangements are there? And is there an arrangement that is optimum according to some criterion? We will focus mainly on the principles that guarantee existence, and the techniques of enumeration, and also give a few examples concerning optimization.

Calendar Description:

MATH 421 Combinatorics

Permutations and Combinations, Binomial Theorem, Principle of Inclusion-Exclusion, recurrence relations, generating functions, orthogonal Latin squares, balanced incomplete block designs, Steiner triple systems, perfect difference sets, Boolean algebra and Finite State Machines. Prerequisites: MATH 228 (or 223 or 128); any 300 - level MATH course, MATH 322 recommended.

Policies:

Policy about course outlines can be found in Section 23.4(2) of the University Calendar.

Code of Student Behavior:

The University of Alberta is committed to the highest standards of academic integrity and honesty. Students are expected to be familiar with these standards regarding academic honesty and to uphold the policies of the University in this respect. Students are particularly urged to familiarize themselves with the provisions of the Code of Student Behavior

(online at http://www.ualberta.ca/secretariat/appeals.htm)

and avoid any behavior which could potentially result in suspicion of cheating, plagiarism, misrepresentation of facts and/or participation in an offence. Academic dishonesty is a serious offence and can result in suspension or expulsion from the University.

Help:

The Department of Mathematical and Statistical Sciences runs a free "walk-in" math help clinic every week day from 9am until 3pm in ED 751.

The Mathematics and Applied Sciences Centre, located in E6-050 of ETLC, provides an extra help program. Their phone number is 492-6272 and their email address is MASC@ualberta.ca They offer tutorial sessions and midterm and final exam prep classes.

Students with learning disabilities will find that the University has a friendly, well organized office dealing with these issues. Their web site is http://www.ualberta.ca/ssds.

Topics to be Covered:

I. Combinations

- 2.1 The Product Rule
- 2.2 The Sum Rule
- 2.6 Subsets
- 2.7 r-Combinations
- 2.8 Probability
- 2.9 Sampling with Replacement
- 2.10 Occupancy Problems
  - 2.10.1 The Types of Occupancy Problems
  - 2.10.3 Indistinguishable Balls and Distinguishable Cells
  - 2.10.5 Indistinguishable Balls and Indistinguishable Cells
- 2.14 The Binomial Expansion
- 7.1 The Principle of Inclusion and Exclusion and some of its Applications
  - 7.1.1 Some Simple Examples
  - 7.1.2 Proof of Inclusion and Exclusion Principle
  - 7.1.3 Prime Numbers, Cryptography, and Sieves
  - 7.1.4 The Probabilistic Case
  - 7.1.5 The Occupancy Problem with Distinguishable Balls and Cells
  - 7.1.8 Counting Combinations
- 7.2 The Number of Objects Having Exactly m Properties

II. Algebraic Techniques: Generating Functions and Recurrence Relations

- 5.1 Example of Generating Functions
- 5.2 Operating on Generating Functions
- 5.3 Applications to Counting
- 5.4 The Binomial Theorem
- 6.1 Some Examples of Recurrence Relations
  - 6.1.1 Some Simple Recurrences
  - 6.1.2 Fibonacci Numbers and Their Applications
  - 6.1.4 Recurrences Involving More Than One Sequence
- 6.2 The Method of Characteristic Roots
- 6.3 Solving Recurrences Using Generating Functions
  - 6.3.1 The Method
  - 6.3.3 Simultaneous Equations for Generating Functions
- 6.4 Some Recurrences Involving Convolutions
  - 6.4.1 The Number of Simple, Ordered, Rooted Trees
  - 6.4.2 The Number of Ways to Multiply a Sequence of Numbers in a Computer

III. Permutations

- 2.3 Permutations
- 2.5 r-Permutations
- 2.10 Occupancy Problems
  - 2.10.1 The Types of Occupancy Problems
  - 2.10.2 Distinguishable Balls and Distinguishable Cells
- 2.13 Permutations with Classes of Indistinguishable Objects Revisited

- 5.5 Exponential Generating Functions and Generating Functions for Permutations
- 6.1 Some Examples of Recurrence Relations
  - 6.1.3 Derangements
- 6.3 Solving Recurrences Using Generating Functions
  - 6.3.2 Derangements
- 7.1 The Principle of Inclusion and Exclusion and Some of its Applications
  - 7.1.7 Derangements
- 7.2 The Number of Objects Having Exactly m Properties
  - 7.2.1 The Main Result and Its Applications

IV. Symmetries: The Pólya Theory of Counting

- 8.1 Equivalence Relations
- 8.2 Permutation Groups
  - 8.2.1 Definition of a Permutation Group
  - 8.2.2 The Equivalence Relation Induced by a Permutation Group
- 8.3 Burnside's Lemma
- 8.4 Distinct Colorings
  - 8.4.1 Definition of a Coloring
  - 8.4.2 Equivalent Colorings
- 8.5 The Cycle Index
  - 8.5.1 Permutations as Products of Cycles
  - 8.5.2 A Special Case of Pólya's Theorem
  - 8.5.5 The Cycle Index of a Permutation Group
  - 8.5.6 The Proof of a Special Case of Pólya's Theorem
- 8.6 Pólya's Theorem
  - 8.6.1 The Inventory of Colorings
  - 8.6.2 Computing the Pattern Inventory
  - 8.6.4 The Proof of Pólya's Theorem

V. Combinatorial Designs

- 2.19 Pigeonhole Principle and Its Generalizations
  - 2.19.1 The Simplest Version of the Pigeonhole Principle
  - 2.19.2 Generalizations and Applications of the Pigeonhole Principle
- 9.1 Block Designs
- 9.3 Finite Fields and Complete Orthogonal Families of Latin Squares
  - 9.3.1 Modular Arithmetic
  - 9.3.3 The Finite Fields $GF(p^k)$
- 9.4 Balanced Incomplete Block Design
  - 9.4.1 (b, v, r, k, λ)-Designs
  - 9.4.2 Necessary Conditions for the Existence of (b, v, r, k, λ)-Designs
  - 9.4.3 Proof of Fisher's Inequality
  - 9.4.4 Resolvable Designs
  - 9.4.5 Steiner Triple Systems
  - 9.4.6 Symmetric Balanced Incomplete Block Designs
  - 9.4.9 Steiner Systems and the National Lottery
- 9.5 Finite Projective Planes

See the approximate schedule for the due dates for assignments.