

MATH 421 Winter 2006 Combinatorics Assignment 1 Due: Friday January 27, 2006

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## Question 1. [p 111, #4]

How many functions are there each of which assigns a number 0 or a number 1 to each  $m \times n$  matrix of 0's and 1's?

## Question 2. [p 25, #5]

How many 5-letter words either start with a d or do not contain the letter d?

## Question 3. [p 35, #7]

A value function on a set A assigns 0 or 1 to each subset of A.

- (a) If A has 3 elements, how many different value functions are there on A?
- (b) What if A has n elements?

## Question 4. [p 39, #15]

How many odd numbers between 1000 and 9999 have distinct digits?

#### Question 5. [p 40, #19]

Consider the identity

$$\binom{n}{m}\binom{m}{k} = \binom{n}{k}\binom{n-k}{m-k}.$$

- (a) Prove this identity using an "algebraic" proof.
- (b) Prove this identity using a "combinatorial" proof.

## Question 6. [p 46, #10]

- (a) What is the probability that a bit string of length 3, chosen at random, does not have two consecutive 0's?
- (b) What if the bit string has length n?

#### Question 7. [p 51, #6]

Suppose that a codeword of length 8 consists of letters A, B, or C or digits 0 or 1, and cannot start with a 1. How many such code words are there?

## Question 8. [p 57, #1]

Write down all the distributions of:

- (a) 3 distinguishable balls a, b, c into 2 distinguishable cells 1, 2
- (b) 4 distinguishable balls a, b, c, d into 2 distinguishable cells 1, 2
- (c) 2 distinguishable balls a, b into 4 distinguishable cells 1, 2, 3, 4

## Question 9. [p 72, #4]

What is the coefficient of  $x^8$  in the expansion of  $(1+x)^{10}(1+x)^6$ ?

# Question 10. [p73, #5]

Find a formula for  $(a + b + c)^n$ .