MATH 324 Summer 2011
Elementary Number Theory
Assignment 4
Due: Wednesday August 3, 2011

Question 1. [p 222. #14]

Using Fermat's little theorem, find the last digit of the base 7 expansion of 3^{100}.

Question 2. [p 223. #23]

Show that

$$1^{p-1} + 2^{p-1} + 3^{p-1} + \cdots + (p-1)^{p-1} \equiv -1 \pmod{p}$$

whenever p is prime. (It has been conjectured that the converse of this is also true.)

Question 3. [p 223. #28]

Show that if p and q are distinct primes, then

$$p^{q-1} + q^{p-1} \equiv 1 \pmod{pq}.$$

Question 4. [p 223. #41]

(a) Show that if p is a prime, then $\binom{2p}{p} \equiv 2 \pmod{p^2}$.

(b) Can you show that if p is prime, then $\binom{2p}{p} \equiv 2 \pmod{p^3}$?

Question 5. [p 224. #48]

Show that if n is a positive integer with $n \geq 2$, then n does not divide $2^n - 1$.

Question 6. [p 237. #2]

Find a reduced residue system modulo 2^m, where m is a positive integer.

Question 7. [p 237. #6]

Find the last digit of the decimal expansion of $7^{999,999}$.

Question 8. [p 237. #10]

Show that $a^{\phi(b)} + b^{\phi(a)} \equiv 1 \pmod{ab}$ if a and b are relatively prime positive integers.

Question 9. [p 238. #22]

Show that if m is a positive integer, $m > 1$, then $a^m \equiv a^{m-\phi(m)} \pmod{m}$ for all positive integers a.

Question 10. [p 246. #14]

For which positive integers n does $\phi(n) \mid n$?