MATH 324 Summer 2011
Elementary Number Theory
Assignment 3
Due: Wednesday July 27, 2011

Question 1. [p 142. #13]

Which combinations of pennies, dimes, and quarters have a total value of 99¢ ?

Question 2. [p 153. #5]

Show that if a is an odd integer, then $a^2 \equiv 1 \pmod 8$.

Question 3. [p 153. #6]

Find the least nonnegative residue modulo 13 of each of the following integers.

(a) 22 (d) -1

(b) 100 (e) -100

(c) 1001 (f) -1000

Question 4. [p 153. #9]

Find the least positive residue of $1! + 2! + \cdots + 100!$ modulo each of the following integers.

(a) 2 (c) 12

(b) 7 (d) 25

Question 5. [p 154. #29]

For which positive integers n is it true that

$$1^2 + 2^2 + 3^2 + \cdots + (n-1)^2 \equiv 0 \pmod n\,?$$

Question 6. [p 154. #33]

Show that if $n \equiv 3 \pmod 4$, then n cannot be the sum of the squares of two integers.

Question 7. [p 154. #28]

Show that if n is an odd positive integer or if n is a positive integer divisible by 4, then

$$1^3 + 2^3 + 3^3 + \cdots + (n-1)^3 \equiv 0 \pmod n.$$

Is this statement true if n is even but not divisible by 4?

Question 8. [p 161. #18]

Show that if p is an odd prime and a is a positive integer which is not divisible by p, then the congruence $x^2 \equiv a \pmod p$ has either no solution or exactly two incongruent solutions.

Question 9. [p 170. #35]

The three children in a family have feet that are 5 inches, 7 inches, and 9 inches long. When they measure the length of the dining room of their house using their feet, they each find that there are 3 inches left over. How long is the dining room?

Question 10. [p 222. #12]

Using Fermat's little theorem, find the least positive residue of $2^{1000000}$ modulo 17.