MATH 324 Summer 2006
Elementary Number Theory
Assignment 4
Due: Thursday August 10, 2006

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Question 1. **[p 221. #14]**

Using Fermat's little theorem, find the last digit of the base 7 expansion of 3^{100}.

Question 2. **[p 221. #23]**

Show that

$$1^{p-1} + 2^{p-1} + 3^{p-1} + \cdots + (p-1)^{p-1} \equiv -1 \pmod{p}$$

whenever p is prime. (It has been conjectured that the converse of this is also true.)

Question 3. **[p 222. #28]**

Show that if p and q are distinct primes, then

$$p^{q-1} + q^{p-1} \equiv 1 \pmod{pq}.$$

Question 4. **[p 222. #39]**

(a) Show that if p is a prime, then $\binom{2p}{p} \equiv 2 \pmod{p^2}$.

(b) Can you show that if p is prime, then $\binom{2p}{p} \equiv 2 \pmod{p^3}$?

Question 5. **[p 222. #46]**

Show that if n is a positive integer with $n \geq 2$, then n does not divide $2^n - 1$.

Question 6. **[p 236. #2]**

Find a reduced residue system modulo 2^m, where m is a positive integer.

Question 7. **[p 236. #6]**

Find the last digit of the decimal expansion of $7^{999,999}$.

Question 8. **[p 236. #10]**

Show that $a^{\phi(b)} + b^{\phi(a)} \equiv 1 \pmod{ab}$ if a and b are relatively prime positive integers.

Question 9. **[p 236. #20]**

Show that if m is a positive integer, $m > 1$, then $a^m \equiv a^{m-\phi(m)} \pmod{m}$ for all positive integers a.

Question 10. **[p 246. #14]**

For which positive integers n does $\phi(n) \mid n$?