MATH 324 Summer 2006
Elementary Number Theory
Assignment 3
Due: Wednesday August 2, 2006

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Question 1. [p 139. #16]

A piggy bank contains 24 coins, all of which are nickels, dimes, or quarters. If the total value of the coins is two dollars, what combinations of coins are possible?

Question 2. [p 139. #19]

Let a and b be relatively prime positive integers, and let n be a positive integer. A solution (x, y) of the linear diophantine equation $ax + by = n$ is *nonnegative* when both x and y are nonnegative.

Show that whenever $n \geq (a-1)(b-1)$, there is a nonnegative solution of $ax + by = n$.

Question 3. [p 139. #20]

Let a and b be relatively prime positive integers, and let n be a positive integer. Show that if $n = ab - a - b$, then there are no nonnegative solutions of $ax + by = n$.

Question 4. [p 139. #21]

Show that there are exactly $(a-1)(b-1)/2$ nonnegative integers $n < ab - a - b$ such that the equation $ax + by = n$ has a nonnegative solution.

Question 5. [p 150. #17]

What can you conclude if $a^2 \equiv b^2 \pmod{p}$, where a and b are integers and p is a prime?

Question 6. [p 150. #19]

Show that if n is an odd positive integer, then

$$1 + 2 + 3 + \cdots + (n-1) \equiv 0 \pmod{n}.$$

Is this statement true if n is even?

Question 7. [p 150. #20]

Show that if n is an odd positive integer or if n is a positive integer divisible by 4, then

$$1^3 + 2^3 + 3^3 + \cdots + (n-1)^3 \equiv 0 \pmod{n}.$$

Is this statement true if n is even but not divisible by 4?

Question 8. [p 150. #21]

For which positive integers n is it true that

$$1^2 + 2^2 + 3^2 + \cdots + (n-1)^2 \equiv 0 \pmod{n}?$$

Question 9. [p 157. #18]

Show that if p is an odd prime and a is a positive integer which is not divisible by p, then the congruence $x^2 \equiv a \pmod{p}$ has either no solution or exactly two incongruent solutions.

Question 10. [p 167. #33]

The three children in a family have feet that are 5 inches, 7 inches, and 9 inches long. When they measure the length of the dining room of their house using their feet, they each find that there are 3 inches left over. How long is the dining room?