Math 311 - Spring 2014

Assignment # 9

Completion Date: Wednesday June 4, 2014

Question 1. [p 188, #1] Show in two ways that the sequence

$$z_n = -2 + i\,\frac{(-1)^n}{n^2} \qquad (n = 1, 2, \dots)$$

converges to -2.

Question 2. [p 183, Example 2]

Let r_n denote the moduli and Θ_n the principal values of the arguments of the complex numbers z_n in Exercise 1. Show that the sequence r_n $(n = 1, 2, \dots)$ converges but that the sequence Θ_n $(n = 1, 2, \dots)$ does not.

Question 3. [p 188, #3]

Show that

$$\text{if} \qquad \lim_{n\to\infty} z_n = z, \qquad \text{then} \qquad \lim_{n\to\infty} |z_n| = |z|.$$

Question 4. [p 195, #1]

Obtain the Maclaurin series representation

$$z\cosh(z^2) = \sum_{n=0}^{\infty} \frac{z^{4n+1}}{(2n)!} \qquad (|z| < \infty).$$

Question 5. [p 196, #2]

Obtain the Taylor series

$$e^z = e\sum_{n=0}^{\infty} \frac{(z-1)^n}{n!} \qquad (|z-1| < \infty)$$

for the function $f(z) = e^z$ by

(a) using $f^{(n)}(1) \quad (n = 0, 1, 2 \dots)$;

(b) writing $e^z = e^{z-1}e$.

Question 6. [p 196, #3]

Find the Maclaurin series expansion of the function

$$f(z) = \frac{z}{z^4 + 9} = \frac{z}{9} \cdot \frac{1}{1 + (z^4/9)}.$$

Ans: $\sum_{n=0}^{\infty} \frac{(-1)^n}{3^{2n+2}} z^{4n+1} \qquad (|z| < \sqrt{3})$.

Question 7. [p 197, #8]

With the aid of the identity (see Sec. 34)

$$\cos z = -\sin\left(z - \frac{\pi}{2}\right),$$

expand $\cos z$ into a Taylor series about the point $z_0 = \pi/2$.

Question 8. [p 197, #10]

What is the largest circle within which the Maclaurin series for the function $\tanh z$ converges to $\tanh z$? Write the first two nonzero terms of that series.

Question 9. [p 197, #11 (a)]

Show that when $z \neq 0$,

$$\frac{e^z}{z^2} = \frac{1}{z^2} + \frac{1}{z} + \frac{1}{2!} + \frac{z}{3!} + \frac{z^2}{4!} + \cdots.$$

Question 10. [p 206, #5]

Represent the function

$$f(z) = \frac{z+1}{z-1}$$

(a) by its Maclaurin series, and state where the representation is valid;

(b) by its Laurent series in the domain $1 < |z| < \infty$.

Ans: (a) $-1 - 2\sum_{n=1}^{\infty} z^n \quad (|z| < 1)$; (b) $1 + 2\sum_{n=1}^{\infty} \frac{1}{z^n}$.

Question 11. [p 206, #6]

Show that when $0 < |z-1| < 2$,

$$\frac{z}{(z-1)(z-3)} = -3\sum_{n=0}^{\infty} \frac{(z-1)^n}{2^{n+2}} - \frac{1}{2(z-1)}.$$

Question 12. [p 206, #7]

Write the two Laurent series in powers of z that represent the function

$$f(z) = \frac{1}{z(1+z^2)}$$

in certain domains, and specify those domains.

Ans: $\sum_{n=0}^{\infty} (-1)^{n+1} z^{2n+1} + \frac{1}{z} \quad (0 < |z| < 1)$; $\sum_{n=1}^{\infty} (-1)^{n+1} \frac{1}{z^{2n+1}} \quad (1 < |z| < \infty)$.