Math 311 - Spring 2014

Assignment # 6

Completion Date: Friday May 23, 2014

Question 1. [p 109, #9]

With the aid of expressions (15) and (16) in Sec. 34 for $|\sin z|^2$ and $|\cos z|^2$, namely,

$$|\sin z|^2 = \sin^2 x + \sinh^2 y$$

and

$$|\cos z|^2 = \cos^2 x + \sinh^2 y,$$

show that

(a) $|\sinh y| \le |\sin z| \le \cosh y$;

(b) $|\sinh y| \le |\cos z| \le \cosh y$.

Question 2. [p 109, #14]

Show that

(a) $\overline{\cos(i\,z)} = \cos(i\,\overline{z})$ for all z;

(b) $\overline{\sin(i\,z)} = \sin(i\,\overline{z})$ if and only if $z = n\pi i \quad (n = 0, \pm 1, \pm 2, \ldots)$.

Question 3. [p 109, #15]

Find all roots of the equation $\sin z = \cosh 4$ by equating real and imaginary parts of $\sin z$ and $\cosh 4$.

Ans: $\left(\frac{\pi}{2} + 2n\pi\right) \pm 4i \quad (n = 0, \pm 1, \pm 2, \ldots)$.

Question 4. [p 111, #6]

Show that $|\sinh x| \le |\cosh z| \le \cosh x$ by using

(a) identity (12), Sec. 35, namely $|\cosh z|^2 = \sinh^2 x + \cos^2 y$;

(b) the inequalities $|\sinh y| \le |\cos z| \le \cosh y$, obtained in Exercise 9(b), Sec.34.

Question 5. [p 112, #9]

Using the results proved in Exercise 8, locate all zeros and singularities of the hyperbolic tangent function.

Question 6. [p 112, #16]

Find all roots of the equation $\cosh z = -2$. (Compare this exercise with Exercise 16, Sec 34.)

Ans: $\pm \ln(2+\sqrt{3}) + (2n+1)\pi i \quad (n = 0, \pm 1, \pm 2, \ldots)$.

Question 7. [p 114, #2]

Solve the equation $\sin z = 2$ for z by

(a) equating real and imaginary parts in that equation;

(b) Using expression (2), Sec. 36, for $\sin^{-1} z$, namely $\sin^{-1} z = -i \log \left[i\, z + (1 - z^2)^{1/2} \right]$.