Math 311 - Spring 2014

Assignment # 5

Completion Date: Wednesday May 21, 2014

Question 1. [p 97, #1 (b)]

Show that $\text{Log}(1-i) = \dfrac{1}{2}\ln 2 - \dfrac{\pi}{4}\,i.$

Question 2. [p 97, #2 (b)]

Verify that when $n = 0, \pm 1, \pm 2, \ldots,$ $\quad \log i = \left(2n + \dfrac{1}{2}\right)\pi\, i.$

Question 3. [p 97, #2 (c)]

Verify that when $n = 0, \pm 1, \pm 2, \ldots,$ $\quad \log\left(-1 + \sqrt{3}\,i\right) = \ln 2 + 2\left(n + \dfrac{1}{3}\right)\pi\, i.$

Question 4. [p 97, #3]

Show that

(a) $\text{Log}(1+i)^2 = 2\text{Log}(1+i)$; (b) $\text{Log}(-1+i)^2 \neq 2\text{Log}(-1+i)$.

Question 5. [p 97, #5]

Show that

(a) the set of values of $\log\left(i^{1/2}\right)$ is $\left(n + \frac{1}{4}\right)\pi\, i \quad (n = 0, \pm 1, \pm 2, \ldots)$ and that the same is true of $\frac{1}{2}\log i$.

(b) the set of values of $\log\left(i^2\right)$ is *not* the same as the set of values of $2\log i$.

Question 6. [p 97, #9]

Show that

(a) the function $\text{Log}(z-i)$ is analytic everywhere except on the half line $y = 1$ $(x \leq 0)$;

(b) the function

$$\frac{\text{Log}(z+4)}{z^2 + i}$$

is analytic everywhere except at the points $\pm(1-i)/\sqrt{2}$ and on the portion $x \leq -4$ of the real axis.

Question 7. [p 104, #1]

Show that when $n = 0, \pm 1, \pm 2, \ldots,$

(a) $(1+i)^i = \exp\left(-\frac{\pi}{4} + 2n\pi\right) \exp\left(\frac{i}{2} \ln 2\right);$

(b) $(-1)^{1/\pi} = e^{(2n+1)i}.$

Question 8. [p 104, #2 (c)]

Find the principal value of $(1-i)^{4i}$.