Math 311 - Spring 2014

Assignment # 2

Completion Date: Friday May 9, 2014

Question 1. [p 29, #2]

In each case, find all of the roots in rectangular coordinates, exhibit them as vertices of certain squares, and point out which is the principal root:

(a) $(-16)^{1/4}$; (b) $\left(-8-8\sqrt{3}\,i\right)^{1/4}$.

Ans: (a) $\pm\sqrt{2}\,(1+i)$, $\pm\sqrt{2}\,(1-i)$; (b) $\pm(\sqrt{3}-i)$, $\pm(1+\sqrt{3}\,i)$.

Question 2. [p 30, #3]

In each case, find all of the roots in rectangular coordinates, exhibit them as vertices of certain regular polygons, and identify the principal root:

(a) $(-1)^{1/3}$; (b) $8^{1/6}$.

Ans: (b) $\pm\sqrt{2}$, $\pm\dfrac{1+\sqrt{3}\,i}{\sqrt{2}}$, $\pm\dfrac{1-\sqrt{3}\,i}{\sqrt{2}}$.

Question 3. [p 33, #1]

Sketch the following sets and determine which are domains:

(a) $|z-2+i|\le 1$; (b) $|2z+3|>4$; (c) $\text{Im } z>1$;
(d) $\text{Im } z=1$; (e) $0\le \arg z\le \pi/4$ $(z\neq 0)$; (f) $|z-4|\ge |z|$.

Ans: (b), (c) are domains.

Question 4. [p 33, #4]

In each case, sketch the closure of the set:

(a) $-\pi<\arg z<\pi$ $(z\neq 0)$; (b) $|\text{Re } z|<|z|$;
(c) $\text{Re}\left(\dfrac{1}{z}\right)\le \dfrac{1}{2}$; (d) $\text{Re}\left(z^2\right)>0$.

Question 5. [p 37, #2]

Write the function $f(z)=z^3+z+1$ in the form $f(z)=u(x,y)+i\,v(x,y)$.

Ans: $(x^3-3xy^2+x+1)+i\,(3x^2y-y^3+y)$.

Question 6. [p 37, #3]

Suppose that $f(z) = x^2 - y^2 - 2y + i\,(2x - 2xy)$, where $z = x + i\,y$. Use the expressions (see Sec. 5)

$$x = \frac{z + \overline{z}}{2} \qquad \text{and} \qquad y = \frac{z - \overline{z}}{2i}$$

to write $f(z)$ in terms of z and simplify the result.

Ans: $\overline{z}^2 + 2i\,z$.

Question 7. [p 44, #1]

By referring to Example 1 in Sec. 13, find a domain in the z plane whose image under the transformation $w = z^2$ is the square domain in the w plane bounded by the lines $u = 1$, $u = 2$, $v = 1$, and $v = 2$. (See Fig. 2, Appendix 2.)

Question 8. [p 44, #3]

Sketch the region onto which the sector $r \leq 1$, $0 \leq \theta \leq \pi/4$ is mapped by the transformation

(a) $w = z^2$; (b) $w = z^3$; (c) $w = z^4$.