Math 311 - Spring 2014

Assignment # 12

Completion Date: Thursday June 12, 2014

Question 1. [p 267, #2]

Use residues to evaluate the improper integral

$$\int_0^\infty \frac{dx}{(x^2+1)^2}.$$

Ans: $\pi/4$.

Question 2. [p 267, #5]

Use residues to evaluate the improper integral

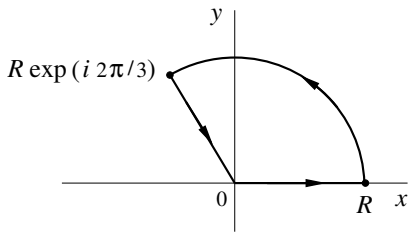
$$\int_0^\infty \frac{x^2\,dx}{(x^2+9)(x^2+4)^2}.$$

Ans: $\pi/200$.

Question 3. [p 267, #8]

Use residues and the contour shown in Fig. 95, where $R > 1$, to establish the integration formula

$$\int_0^\infty \frac{dx}{x^3+1} = \frac{2\pi}{3\sqrt{3}}.$$



Question 4. [p 276, #5]

Use residues to evaluate the improper integral

$$\int_{-\infty}^\infty \frac{x\sin ax}{x^4+4}\,dx \quad (a>0).$$

Ans: $\dfrac{\pi}{2}e^{-a}\sin a$.

Question 5. [p 276, #9]

Use residues to find the Cauchy principal value of the improper integral

$$\int_{-\infty}^{\infty} \frac{\sin x \, dx}{x^2 + 4x + 5}.$$

Ans: $-\dfrac{\pi}{e} \sin 2.$

Question 6. [p 286, #2]

Evaluate the improper integral

$$\int_0^{\infty} \frac{x^a}{(x^2+1)^2} \, dx, \quad \text{where} \quad -1 < a < 3 \quad \text{and} \quad x^a = \exp(a \ln x).$$

Ans: $\dfrac{(1-a)\pi}{4\cos(a\pi/2)}.$

Question 7. [p 290, #2]

Use residues to evaluate the definite integral

$$\int_{-\pi}^{\pi} \frac{d\theta}{1 + \sin^2 \theta}.$$

Ans: $\sqrt{2}\,\pi.$

Question 8. [p 291, #5]

Use residues to evaluate the definite integral

$$\int_0^{\pi} \frac{\cos 2\theta \, d\theta}{1 - 2a\cos\theta + a^2} \quad (-1 < a < 1).$$

Ans: $\dfrac{a^2\pi}{1-a^2}.$

Question 9. [p 291, #6]

Use residues to evaluate the definite integral

$$\int_0^{\pi} \frac{d\theta}{(a + \cos\theta)^2} \quad (a > 1).$$

Ans: $\dfrac{a\pi}{\left(\sqrt{a^2-1}\right)^3}.$