Math 311 - Spring 2014

Assignment # 10

Completion Date: Friday June 6, 2014

Question 1. [p 219, #1]

By differentiating the Maclaurin series representation

$$\frac{1}{1-z} = \sum_{n=0}^{\infty} z^n \qquad (|z| < 1),$$

obtain the expansions

$$\frac{1}{(1-z)^2} = \sum_{n=0}^{\infty} (n+1) z^n \qquad (|z| < 1)$$

and

$$\frac{2}{(1-z)^3} = \sum_{n=0}^{\infty} (n+1)(n+2) z^n \qquad (|z| < 1).$$

Question 2. [p 219, #2]

By substituting $1/(1-z)$ for z in the expansion

$$\frac{1}{(1-z)^2} = \sum_{n=0}^{\infty} (n+1) z^n \qquad (|z| < 1),$$

found in Exercise 1, derive the Laurent series representation

$$\frac{1}{z^2} = \sum_{n=2}^{\infty} \frac{(-1)^n (n-1)}{(z-1)^n} \qquad (1 < |z-1| < \infty).$$

(Compare Example 2, Sec. 65.)

Question 3. [p 220, #3]

Find the Taylor series for the function

$$\frac{1}{z} = \frac{1}{2+(z-2)} = \frac{1}{2} \cdot \frac{1}{1+(z-2)/2}$$

about the point $z_0 = 2$. Then by differentiating that series term by term, show that

$$\frac{1}{z^2} = \frac{1}{4} \sum_{n=0}^{\infty} (-1)^n (n+1) \left(\frac{z-2}{2}\right)^n \qquad (|z-2| < 2).$$

Question 4. [p 215, Example 1.]

With the aid of series, prove that the function f defined by means of the equations

$$f(z) = \begin{cases} \dfrac{e^z - 1}{z} & \text{when} \quad z \neq 0, \\ 1 & \text{when} \quad z = 0 \end{cases}$$

is entire.

Question 5. [p 225, #1]

Use multiplication of series to show that

$$\frac{e^z}{z(z^2+1)} = \frac{1}{z} + 1 - \frac{1}{2}\,z - \frac{5}{6}\,z^2 + \cdots \qquad (0 < |z| < 1).$$

Question 6. [p 225, #2]

By writing $\csc z = 1/\sin z$ and then using division, show that

$$\csc z = \frac{1}{z} + \frac{1}{3!}\,z + \left[\frac{1}{(3!)^2} - \frac{1}{5!}\right] z^3 + \cdots \qquad (0 < |z| < \pi).$$