Math 309 - Spring-Summer 2017

Problem Set # 6

Completion Date: Friday June 16, 2017

Question 1.

With the aid of expressions,

$$|\sin z|^2 = \sin^2 x + \sinh^2 y$$

and

$$|\cos z|^2 = \cos^2 x + \sinh^2 y,$$

show that

(a) $|\sinh y| \leq |\sin z| \leq \cosh y$;

(b) $|\sinh y| \leq |\cos z| \leq \cosh y$.

Question 2.

Show that

(a) $\overline{\cos(i\,z)} = \cos(i\,\overline{z})$ for all z;

(b) $\overline{\sin(i\,z)} = \sin(i\,\overline{z})$ if and only if $z = n\pi i \quad (n = 0, \pm 1, \pm 2, \dots)$.

Question 3.

Find all roots of the equation $\sin z = \cosh 4$ by equating real and imaginary parts of $\sin z$ and $\cosh 4$.

Ans: $\left(\dfrac{\pi}{2} + 2n\pi\right) \pm 4i \quad (n = 0, \pm 1, \pm 2, \dots)$.

Question 4.

Show that $|\sinh x| \leq |\cosh z| \leq \cosh x$ by using

(a) the identity $|\cosh z|^2 = \sinh^2 x + \cos^2 y$;

(b) the inequalities $|\sinh y| \leq |\cos z| \leq \cosh y$.

Question 5.

Locate all zeros and singularities of the hyperbolic tangent function.

Question 6.

Find all roots of the equation $\cosh z = -2$.

Ans: $\pm \ln(2+\sqrt{3}) + (2n+1)\pi i \quad (n = 0, \pm 1, \pm 2, \ldots)$.

Question 7.

Solve the equation $\sin z = 2$ for z by

(a) equating real and imaginary parts in that equation;

(b) using the expression $\sin^{-1} z = -i \log \left[i\, z + (1 - z^2)^{1/2} \right]$.