Math 309 - Spring-Summer 2017

Problem Set # 5

Completion Date: Friday June 9, 2017

Question 1.

Show that $\mathrm{Log}(1 - i) = \dfrac{1}{2}\ln 2 - \dfrac{\pi}{4}\, i$.

Question 2.

Verify that when $n = 0, \pm 1, \pm 2, \ldots,$ $\log i = \left(2n + \dfrac{1}{2}\right)\pi\, i$.

Question 3.

Verify that when $n = 0, \pm 1, \pm 2, \ldots,$ $\log\left(-1 + \sqrt{3}\, i\right) = \ln 2 + 2\left(n + \dfrac{1}{3}\right)\pi\, i$.

Question 4.

Show that

(a) $\mathrm{Log}(1 + i)^2 = 2\mathrm{Log}(1 + i)$;   (b) $\mathrm{Log}(-1 + i)^2 \neq 2\mathrm{Log}(-1 + i)$.

Question 5.

Show that

(a) the set of values of $\log\left(i^{1/2}\right)$ is $\left(n + \frac{1}{4}\right)\pi\, i$ $(n = 0, \pm 1, \pm 2, \ldots)$ and that the same is true of $\frac{1}{2}\log i$.

(b) the set of values of $\log\left(i^2\right)$ is *not* the same as the set of values of $2\log i$.

Question 6.

Show that

(a) the function $\mathrm{Log}(z - i)$ is analytic everywhere except on the half line $y = 1$ $(x \leq 0)$;

(b) the function

$$\frac{\mathrm{Log}(z + 4)}{z^2 + i}$$

is analytic everywhere except at the points $\pm(1 - i)/\sqrt{2}$ and on the portion $x \leq -4$ of the real axis.

Question 7.

Show that when $n = 0, \pm 1, \pm 2, \ldots$,

(a) $(1+i)^i = \exp\left(-\dfrac{\pi}{4} + 2n\pi\right) \exp\left(\dfrac{i}{2} \ln 2\right)$;

(b) $(-1)^{1/\pi} = e^{(2n+1)i}$.

Question 8.

Find the principal value of $(1 - i)^{4i}$.