Math 309 Spring-Summer 2017

Mathematical Methods for Electrical Engineers

Topics for Final Examination

Department of Mathematical and Statistical Sciences

University of Alberta

Date: July 31, 2017

The final examination will be held in **ETLC E2-001 on Tuesday, August 15, 2017, from 9:00 am until 11:00 am**, and may include questions from **all** of the course material covered during the term.

1. Number systems $\mathbb{N}$, $\mathbb{Z}$, $\mathbb{Q}$, and $\mathbb{R}$.
2. Complex numbers $\mathbb{C}$, addition and multiplication and field axioms.
3. Conjugate and Modulus of complex numbers, triangle inequality.
4. Polar form of complex numbers, arg and Arg, DeMoivre's theorem, Euler's formulas.
5. Powers and roots, principal root.
6. Topology in $\mathbb{C}$: neighborhoods, deleted neighborhoods, open sets, connected sets, domains.
7. Functions of a complex variable $f : \mathbb{C} \longrightarrow \mathbb{C}$, $f(z) = u(x, y) + iv(x, y)$ as a mapping from the plane into the plane, the exponential function e^z.
8. Definition of limit, properties of limits, limits of functions $f(z)$ in terms of limits of the real and imaginary parts of $f(z)$.
9. Examples of limits of functions like $\text{Arg}(z)$, $\dfrac{\overline{z}}{z}$, $\text{Re}(z)$, Im(z), $\overline{z}$, $|z|$, as $z \to 0$ or $z \to z_0$.
10. Continuity and differentiablity, differentiability implies continuity.
11. Derivative, theorems for derivative: product rule, quotient rule, chain rule ...
12. Cauchy-Riemann equations, complex differentiability equivalent to real differentiability plus Cauchy-Riemann equations.
13. Cauchy-Riemann equations in polar coordinates, for example $\text{Log}(z) = \ln |z| + i\text{Arg}\, z$.
14. A function $f(z)$ is analytic at a point z_0 if and only if it is differentiable at every point in some neighborhood of z_0.
15. Analytic functions, harmonic functions, hamonic conjugates.
16. Trigonometric functions: $\sin z$, $\cos z$, $\tan z$, etc., properties.
17. Hyperbolic trigonometric functions: $\sinh z$, $\cosh z$, $\tanh z$, etc., properties.
18. Logarithmic Functions: $\log z$ and $\text{Log}\, z$. Powers z^α where $\alpha \in \mathbb{C}$.
19. Properties of logarithms, e.g. $e^{\log z} = z$, but $\log e^z \neq z$.
20. Inverse trigonometric functions, inverse hyperbolic trigonometric functions.

21. Integrals of complex valued functions of a real variable:

$$\int_a^b [u(t) + iv(t)]\, dt = \int_a^b u(t)\, dt + i \int_a^b v(t)\, dt,$$

for example, Fourier coefficients $\displaystyle\int_{-\pi}^{\pi} f(t)e^{int}\, dt$.

22. Properties of integrals of complex valued functions of a real variable, for example,

$$\left| \int_a^b f(t)\, dt \right| \leq \int_a^b |f(t)|\, dt.$$

23. Curves in the complex plane, smooth arcs, arc length, line integrals or contour integrals.

24. Properties of contour integrals, especially

$$\left| \int_C f(z)\, dz \right| \leq M \cdot L.$$

25. Equivalence of: antiderivatives on $\mathbb{C}$, independence of path for contour integrals, integrals around simple closed contours are zero.

26. Cauchy-Goursat theorem (using Green's Theorem in the plane and the Cauchy-Riemann equations).

27. The Cauchy integral theorem, Cauchy integral theorem for derivatives, Morera's theorem.

28. Infinite sequences and series in $\mathbb{C}$, for example, geometric series.

29. Taylor series (with remainder) proved using geometric series and Cauchy integral formula.

30. Laurent series, the principal part of a function.

31. Residues and the Cauchy residue theorem.

32. Examples of the residue theorem, types of singular points.

33. Residues at poles, applications to real improper integrals (Fourier integrals)

$$\int_{-\infty}^{\infty} f(x) \sin ax\, dx \qquad \text{and} \qquad \int_{-\infty}^{\infty} f(x) \cos ax\, dx,$$

34. Jordan's Inequality and Jordan's Lemma.

35. Applications of residue theorem to integrals of the form

$$\int_0^{2\pi} F(\sin\theta, \cos\theta)\, d\theta.$$

36. Applications of residue theorem to integrals over indented contours.

37. Applications of residue theorem to integrals of multiple valued functions.

38. Bromwich's Integral (time permitting).

Sections from the Text:

Chap. 1. Sections: 1.1, 1.2, 1.3, 1.4, 1.5, 1.6, 1.7

Chap. 2. Sections: 2.1, 2.2, 2.3, 2.4, 2.5

Chap. 3. Sections: 3.1, 3.2, 3.3, 3.4, 3.5

Chap. 4. Sections: 4.1, 4.2, 4.3, 4.4a, 4.4b, 4.5, 4.6, 4.7

Chap. 5. Sections: 5.1, 5.2, 5.3, 5.4, 5.5, 5.6, 5.7

Chap. 6. Sections: 6.1, 6.2, 6.3, 6.4, 6.5, 6.6

Chap. 8. Sections: 8.3