

Midterm I:

June 6 9:00 - 9:50 in CAB 243

(No electronics, bring OneCard)

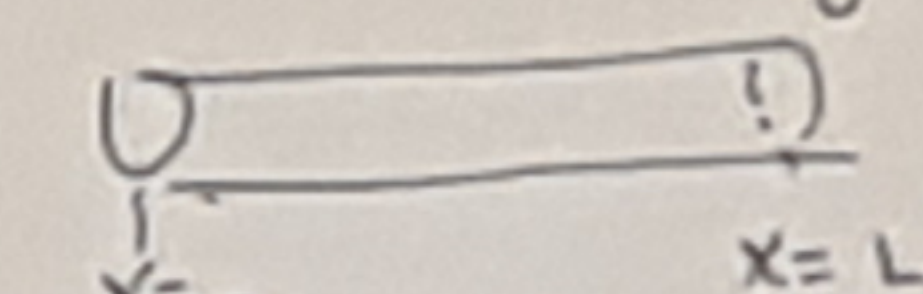
3 Problems: { Separation of Variables
Fourier Series
PDEs

Start Fourier Series on Friday } Chap 2
May 25 - June 4

Essential Steps for Separation:

Running Example: Heat Eqn in rod of length L

$$\frac{\partial u}{\partial t} = k \frac{\partial^2 u}{\partial x^2} \quad \left\{ \text{PDE} \right.$$



$$u(0,t) = 0 \quad \text{for all } t > 0$$

$$u(L,t) = 0 \quad \text{for all } t > 0$$

} B.C.

$$u(x,0) = f(x), \quad 0 \leq x \leq L \quad \left\{ \text{I.C.} \right.$$

Boundary Value - Initial Value Problem
(BV-IVP)

1. Separation: Only works when
B.C. are linear and homogeneous
P.D.E. is linear and homogeneous

Assume a soln of the form

$$u(x,t) = \phi(x) \cdot G(t)$$

2. Substitute this into the PDE
and the homogeneous part of boundary
and initial conditions, and derive two
ordinary differential eqns for $\phi(x)$ and $G(t)$

$$\varphi(x) G'(t) = k \varphi'(x) G(t)$$

$$\frac{\varphi'(x)}{\varphi(x)} = \frac{G'(t)}{k G(t)} \quad (*)$$

↑
depends
only on x
not t .

↑
depends
only on t
not on x

Since x and t are independent variables

So from $(*)$ $\frac{\varphi'(x)}{\varphi(x)} = -\lambda = \frac{G'(t)}{k G(t)}$

Ord. diff. eqns.

x -eqn:

$$\varphi''(x) + \lambda \varphi(x) = 0, \quad 0 \leq x \leq L$$

$$\varphi(0) = \varphi(L) = 0$$

t -eqn:

$$G'(t) + \lambda k G(t) = 0, \quad t > 0$$

3. Identify the complete problem of these two (the one with the correct number of side conditions so we can solve it).

e.g. we get a nontrivial solution to the x -eqn for those values of λ such that

$$\lambda = \left(\frac{n\pi}{L}\right)^2 \leftarrow \text{eigenvalues}$$

with corresponding eigenfunctions

$$\varphi_n(x) = \sin\left(\frac{n\pi}{L}x\right) \leftarrow \text{eigenfunctions}$$

4. Use those values of λ to solve the time problem:

$$G_n'(t) + \lambda k G_n(t) = 0$$

$$G_n(t) = z_n e^{-\lambda k t} \quad t > 0$$

5. Use superposition principle

$$u(x,t) = \sum_{n=1}^{\infty} a_n e^{-k\left(\frac{n\pi}{L}\right)^2 t} \sin\left(\frac{n\pi}{L}x\right) \quad 0 \leq x \leq L, t > 0$$

Satisfies PDE + the BCs but at the initial condition

4 Use those values of λ_n
to solve the time problem:

$$G_n'(t) + k\lambda_n G_n(t) = 0$$

$$\underline{G_n(t) = Z_n e^{-k\lambda_n t}, \quad t \geq 0}$$

5 Use superposition principle

$$u(x,t) = \sum_{n=1}^{\infty} a_n e^{-k\left(\frac{n\pi}{L}\right)^2 t} \sin\left(\frac{n\pi}{L}x\right), \quad 0 \leq x \leq L, \quad t \geq 0$$

Satisfies PDE & the BC, but not the initial
Condition

6 Use the inhomogeneous Side Condition by
Setting $t=0$ to get

$$u(x,0) = f(x) = \sum_{n=1}^{\infty} a_n \sin\left(\frac{n\pi}{L}x\right)$$

7 Find the Fourier Sine Series of $f(x)$
to get the constants a_n .

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