MATH 243 Winter 2008
Geometry II: Transformation Geometry
Problem Set 5
Completion Date: Friday April 11, 2008

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Question 1. *Thomsen's Relation* Prove that for any lines a, b, c :

$$\sigma_c\,\sigma_a\,\sigma_b\,\sigma_c\,\sigma_a\,\sigma_b\,\sigma_a\,\sigma_b\,\sigma_c\,\sigma_a\,\sigma_b\,\sigma_c\,\sigma_b\,\sigma_a\,\sigma_c\,\sigma_b\,\sigma_a\,\sigma_b\,\sigma_a\,\sigma_c\,\sigma_b\,\sigma_a = \iota.$$

Question 2. If $x' = ax + by + c$ and $y' = bx - ay + d$ with $a^2 + b^2 = 1$ are the equations for an isometry α, show that α is a reflection if and only if

$$ac + bd + c = 0 \qquad \text{and} \qquad ad - bc - d = 0.$$

Question 3. If $x' = \frac{3}{5}x + \frac{4}{5}y$ and $y' = \frac{4}{5}x - \frac{3}{5}y$ are the equations for σ_m, then find the line m.

Question 4. If $2x' = -\sqrt{3}x - y + 2$ and $2y' = x - \sqrt{3}y - 1$ are the equations for $\rho_{P,\theta}$, then find P and θ.

Question 5. If $x' = ax + by + c$ and $y' = bx - ay + d$ are equations for σ_m, then find the line m.

Question 6. Show that the equations for a glide reflection whose axis m passes through the origin with angle of inclination θ and whose translation is along m through r units, r measured positive from the origin into the first two quadrants or along the positive x-axis, and negative otherwise, are given by

$$x' = x\cos 2\theta + y\sin 2\theta + r\cos\theta$$
$$y' = x\sin 2\theta - y\cos 2\theta + r\sin\theta.$$

Question 7. If a and b are lines in the plane, show that the following are equivalent:

(a) $a = b$ or a and b are perpendicular,

(b) $\sigma_a\sigma_b = \sigma_b\sigma_a$,

(c) $\sigma_b(a) = a$,

(d) $(\sigma_b\sigma_a)^2 = \iota$,

(e) $\sigma_b\sigma_a$ is either the identity or a halfturn.

Question 8. If the isometry σ_P is a halfturn, show that given any two perpendicular lines m and n which intersect at the point P, we have $\sigma_P = \sigma_m\sigma_n$.