MATH 243 Winter 2008
Geometry II: Transformation Geometry
Problem Set 4
Completion Date: Monday March 17, 2008

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Question 1. If ℓ, m, and n are the perpendicular bisectors of the sides $[AB]$, $[BC]$, and $[CA]$, respectively, of $\triangle ABC$, then

$$\alpha = \sigma_n \sigma_m \sigma_\ell$$

is a reflection in which line?

Question 2. If $\sigma_n \sigma_m \sigma_\ell$ is a reflection, show that the lines ℓ, m, n are concurrent or have a common perpendicular.

Question 3. Find equations for lines m and n such that

$$\sigma_m \sigma_n (x, y) = (x + 2, y - 4).$$

Question 4. Show that

$$\sigma_P \sigma_\ell \sigma_P \sigma_\ell \sigma_P \sigma_\ell \sigma_P$$

is a reflection in a line parallel to ℓ.

Question 5. Let C be a point on the line ℓ, show that

$$\sigma_\ell \rho_{C,\theta} \sigma_\ell = \rho_{C,-\theta}.$$

Question 6. Given nonparallel lines AB and CD, show that there is a rotation ρ such that

$$\rho(AB) = CD.$$

Question 7. Show that if ρ_1, $\rho_2, \rho_2\rho_1$, and $\rho_2^{-1}\rho_1$ are rotations, then the center of ρ_1, $\rho_2\rho_1$, and $\rho_2^{-1}\rho_1$ are collinear.

Question 8. In a given acute triangle, inscribe a triangle $\triangle PQR$ having minimum perimeter.

Question 9. Given a point A and two lines ℓ and m, construct a square $ABCD$ such that B lies on ℓ and D lies on m.

Question 10. Given four distinct points, find a square such that each of the lines containing a side of the square passes through one of the four given points.

Hint: Given A, B, C, D we want to find the lines a, b, c, and d. Take P such that $\rho_{P,90}(B) = C$. Let $\rho_{P,90}(D) = E$, then take a to be AE.