MATH 243 Winter 2008
Geometry II: Transformation Geometry
Problem Set 3
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Department of Mathematical and Statistical Sciences
University of Alberta

Question 1. Given a point O and a vector $\overrightarrow{u}$.

(a) Find the point Q such that
$$\tau_{\overrightarrow{u}}\sigma_O\tau_{\overrightarrow{u}}^{-1} = \sigma_Q.$$

(b) What is the product $\sigma_O\tau_{\overrightarrow{u}}$?

Question 2. In the triangle $\triangle ABC$, show that G is the centroid if and only if
$$\sigma_G\sigma_C\sigma_G\sigma_B\sigma_G\sigma_A = \iota.$$

Question 3. Prove using halfturns, that if $ABCD$ and $EBFD$ are parallelograms, then $EAFC$ is also a parallelogram.

Question 4. Find all triangles such that three given noncollinear points are the midpoints of the sides of the triangle.

Hint: Given P, Q, R then $\sigma_R\sigma_Q\sigma_P$ fixes a vertex of a unique triangle $\triangle P'Q'R'$.

Question 5. Given $\angle ABC$, construct a point P on AB and a point Q on BC such that $PQ = AB$ and the line PQ intersects the line BC at an angle of $60°$.

Hint: Take a point D such that $[BD] \equiv [AB]$ and BD intersects BC at an angle of $60°$.

Question 6. What is the symmetry group of a rhombus that is not a square?

Question 7. Prove that if $\sigma_n\sigma_m$ fixes the point P and $m \neq n$, then the point P is on both lines m and n.

Question 8. Let m be a line with equation $2x + y = 1$. Find the equations of σ_m.

Question 9. Suppose that the lines ℓ and m have equations $x + y = 0$ and $x - y = 1$, respectively. Find the equations for $\sigma_\ell\sigma_m$.

Question 10. Given triangles $\triangle ABC$ and $\triangle DEF$, where $\triangle ABC \equiv \triangle DEF$ where
$$A(0,0),\ B(5,0),\ C(0,10),\ D(4,2),\ E(1,-2),\ F(12,-4),$$
find the equations of the lines such that the product of reflections in these lines maps $\triangle ABC$ to $\triangle DEF$.