math 228

Assignment 3, due Wednesday May 23, 2007

Question 1. [Exercises 3.1, # 2]

Let $R = \{0,\, e,\, b,\, c\}$ with addition and multiplication defined by the following tables. Assume associativity and distributivity and show that R is a ring with identity. Is R commutative? Is R a field?

$+$	0	e	b	c
0	0	e	b	c
e	e	0	c	b
b	b	c	0	e
c	c	b	e	0

$\cdot$	0	e	b	c
0	0	0	0	0
e	0	e	b	c
b	0	b	b	0
c	0	c	0	c

Question 2. [Exercises 3.1, # 10]

Let $\mathbb{Z}[\,i\,]$ denote the set $\{a + b\,i \mid a, b \in \mathbb{Z}\}$. Show that $\mathbb{Z}[\,i\,]$ is a subring of $\mathbb{C}$.

Question 3. [Exercises 3.1, # 12].

Let T be the ring of continuous functions from $\mathbb{R}$ to $\mathbb{R}$. Let

$$S = \{f \in T \mid f(2) = 0\}.$$

Is S a subring of T?

Question 4. [Exercises 3.1, # 16].

Show that the subset $R = \{\,0,\, 3,\, 6,\, 9,\, 12,\, 15\,\}$ of $\mathbb{Z}_{18}$ is a subring. Does R have an identity?

Question 5. [Exercises 3.1, # 18].

Define a new addition $\oplus$ and multiplication $\odot$ on $\mathbb{Z}$ by

$$a \oplus b = a + b - 1 \qquad \text{and} \qquad a \odot b = a + b - ab,$$

where the operations on the right-hand side of the equal signs are ordinary addition, subtraction, and multiplication. Prove that with the new operations $\oplus$ and $\odot$, $\mathbb{Z}$ is an integral domain.

Question 6. [Exercises 3.1, # 24].

The addition table and part of the multiplication table for a three-element ring are given below. Use the distributive laws to complete the multiplication table.

$+$	r	s	t
r	r	s	t
s	s	t	r
t	t	r	s

$\cdot$	r	s	t
r	r	r	r
s	r	t	
t	r		

Question 7. [Exercises 3.2, # 2].

An element e of a ring is said to be **idempotent** if $e^2 = e$.

(a) Find four idempotent elements in the ring $M(\mathbb{R})$.

(b) Find all idempotents in $\mathbb{Z}_{12}$.

(c) Prove that the only idempotents in an integral domain are O_R and 1_R.

Question 8. [Exercises 3.2, # 12].

(a) Prove that $[\,a\,]$ is a unit in $\mathbb{Z}_n$ if and only if $(a, n) = 1$ in $\mathbb{Z}$.

(b) Prove that $[\,a\,]$ is a nonunit in $\mathbb{Z}_n$ if and only if $[\,a\,]$ is a zero divisor.