

---

# math 22

---

## Hints for Assignment 2

**Problem 8.** Twenty girls are sitting at a round table. In front of each are three lights, one green, one red, and one yellow. Each girl is wearing a green hat or a red hat, and each can see all of the hats except her own. The girls are perfect logicians, and they are going to play a game. The object of the game is for each girl to turn on a light that matches the colour of her hat. The game will be played in several rounds. There is a referee who gives a signal to start the round, and at the signal each girl must turn on one of the lights. A girl who does not know the colour of her hat must turn on the yellow light. Before the game begins, the referee tells the girls that at least one of them is wearing a green hat. In fact, ten have green hats and ten have red hats. Explain what happens in each round of the game until all of the girls have determined their hat colours.

Try it first with 4 girls, wearing 2 red hats and 2 green hats:

First Round: All of the girls turn on the yellow light.

Each girl reasons as follows: I see that someone other than me is wearing a green hat, so I cannot conclude anything about the color of my hat. However, if just one girl was wearing a green hat, she would have turned on the green light.

Second Round: The two girls with green hats each turn on the green light, and the two girls with red hats each turn on the yellow light.

Each girl reasons as follows: I know from the first round that there are at least two green hats. If I see two red hats and a green hat, then I realize that the girl wearing the green hat didn't deduce the color of her hat in the first round, so she must have seen a green hat (here it is necessary that she knows that there is at least one green hat, otherwise her logic would fail at the first inductive step) hence I must be wearing a green hat.

If I see two green hats, I cannot deduce that I am wearing a red hat, because the referee did not guarantee at least one red hat.

So at the end of the second round, the two girls with green hats turn on green lights, and the others turn on yellow lights.

Third Round: The two girls with green hats each turn on the green light, and the two girls with red hats each turn on the red light.

Each girl reasons as follows: If I see two green hats, and if there are three green hats (the third being on my head), then in the second round anyone else with a green hat would not have been able to deduce what color hat she had (they would have seen two green hats just like me). Therefore I can conclude that I have a red hat.

So at the end of the third round, the two girls with green hats turn on green lights, and the two girls with red hats turn on red lights.