math22

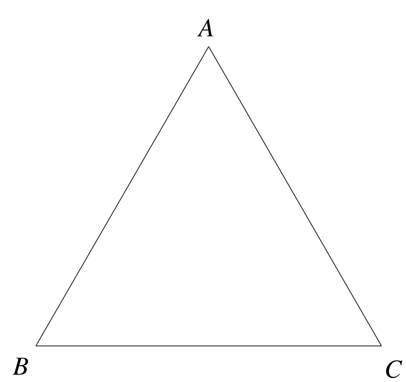
Assignment 2, due Monday February 12, 2007

Problem 1. Consider the following list of binary numbers (it goes on forever):

$$
\begin{array}{c}
1\quad 1 \\
1\quad 0\quad 1 \\
1\quad 0\quad 0\quad 1 \\
1\quad 0\quad 0\quad 0\quad 1 \\
1\quad 0\quad 0\quad 0\quad 0\quad 1 \\
1\quad 0\quad 0\quad 0\quad 0\quad 0\quad 1 \\
1\quad 0\quad 0\quad 0\quad 0\quad 0\quad 0\quad 1 \\
1\quad 0\quad 0\quad 0\quad 0\quad 0\quad 0\quad 0\quad 1 \\
1\quad 0\quad 0\quad 0\quad 0\quad 0\quad 0\quad 0\quad 0\quad 1 \\
.\quad .\quad .\quad .\quad .\quad .\quad .\quad .\quad .\quad .\quad .
\end{array}
$$

Which of these integers is divisible by 3 ? Justify your answer.

Problem 2. Let the triangle ABC be equilateral with $AB = 3$. Show that if we select 10 points in the interior of this triangle, there must be at least two whose distance apart is less than or equal to 1.



Problem 3. The "two-out-of-five" code consists of all possible binary words of length 5 containing exactly two 1's.

(a) List all of the code words.

(b) What is the minimum Hamming distance between code words?

(c) How many errors can the code detect?

(d) How many errors can the code correct?

Problem 4. The following message is received using the 15-digit Hamming Code. Correct the number if it is not correct.

a	a	a	a		a	a	a				a			
b	b	b		b	b			b	b			b		
c	c		c	c		c		c		c			c	
d		d	d	d			d		d	d				d
1	1	1	1	0	0	0	0	0	0	0	1	1	1	1

Problem 5. There are three curtains — labelled A, B, and C. Behind one of them is a brand new BMW, behind each of the others is a goat. Eleven people know what is behind each curtain. You may ask each person one question, which has to have a yes or no answer. Unfortunately, three of them may lie. Devise a set of questions to ask them, and explain clearly why you will be able to tell from the answers where the BMW is.

Problem 6. A message is divided into 6 parts and copies are made. They are dispatched to an agent behind enemy lines by 6 couriers, 2 of whom may be captured.

(a) What is the minimum number of each part that must be carried.

(b) Explain why no courier can carry four parts.

(c) Devise a scheme by which the agent will get at least one copy of each part, while the enemy cannot get copies of all parts.

Problem 7. You have two parents, four grandparents, eight great-grandparents, and so on $\cdots$.

(a) If all of your ancestors were distinct, what would be the total number of your ancestors for the past 40 generations (counting your parents' generation as number one)?

(b) Assuming that each generation represents 30 years, how long is 40 generations?

(c) The total number of people who have ever lived is approximately 10 billion, which equals 10^{10} people. Compare this fact with the answer to part (a). What do you deduce?

Problem 8. Twenty girls are sitting at a round table. In front of each are three lights, one green, one red, and one yellow. Each girl is wearing a green hat or a red hat, and each can see all of the hats except her own. The girls are perfect logicians, and they are going to play a game. The object of the game is for each girl to turn on a light that matches the colour of her hat. The game will be played in several rounds. There is a referee who gives a signal to start the round, and at the signal each girl must turn on one of the lights. A girl who does not know the colour of her hat must turn on the yellow light. Before the game begins, the referee tells the girls that at least one of them is wearing a green hat. In fact, ten have green hats and ten have red hats. Explain what happens in each round of the game until all of the girls have determined their hat colours.

Each girl can actually see that at least one of them is wearing a green hat. So is it necessary for the referee to give them that information?