math22

Assignment 1, due Monday January 29, 2007

Problem 1. Two grade six classes were going on a field trip to a museum. They were being transported by two buses each of which had 34 seats. It so happened that there were 30 boys and 34 girls, and so they put all the boys on one bus and the girls on the other bus. The buses had to stop for a few minutes, and at that time several boys sneaked onto the girls' bus. But the girls' bus driver noticed that there were 10 too many on the bus, so he sent 10 children (boys and girls) back to the boys' bus. After this was done, were there more boys on the girls' bus than girls on the boys' bus? Or vice versa?

Problem 2. There were 5 children in the Emergency Room and between them they had stuck a total of 76 beans up their noses. Show that there must be three children with a combined total of 45 or more beans up their noses.

Problem 3. Dr. Ecco concluded that the aliens from whom the message was received also had an "alphabet" consisting of 26 distinct characters. Another moment's thought convinced him that the code was a linear code with encoding function $E(x) \equiv 5x \pmod{26}$. Your mission, should you decide to accept it, is to help Dr. Ecco find the decoding function $D(x)$, that is, find an integer a such that

$$D(x) \equiv ax \pmod{26},$$

and decode the following message from the aliens: `IH FUAN, KADD JSIU`

Problem 4. Decrypt the following passage which uses a mono-alphabetic code:

E TSWA-EJB-LEFA MAHELSKJNRSP XSLR UKZM JASTRDKZM SN ANNAJLSEH LK LRA QZHH
AJGKUIAJL KQ UKZM TEMBAJ. JAWAM CMSLSCSNA RSN CMAELSWA AQQKMLN, KM SJN-
SNL KJ TSWSJT RSI ZJNKHSCSLAB EBWSCA. XRAJ RA SN SJ LMKZDHA, DA PMAPEMAB LK
RAHP KZL KQ CKZMNA, DZL UKZ NRKZHB MANPACL RSN PMSWECU EL EHH LSIAN. SQ RSN
EJSIEH NRKZHB NLMEU, HAL RSI FJKX DU EHH IAEJN, DZL BKJ'L DA WSJBSCLSWA. EWKSB
CKJNLEJL DKMMKXSJT. E RKZNARKHBAM SN AJLSLHAB LK MAIKWA EJU KDGACL LREL
AJCMKECRAN KJ RSN PMKPAMLU, DZL SL IZNL DA MALZMJAB LK LRA MSTRLQZH KXJAM.

The frequency table is given below:

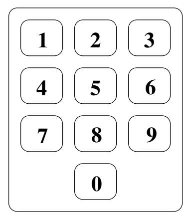
A	B	C	D	E	F	G	H	I	J	K	L	M
48	13	13	11	23	2	2	18	8	28	35	40	27

N	O	P	Q	R	S	T	U	V	W	X	Y	Z
27	0	8	7	21	39	7	9	0	10	5	0	17

Problem 5. The inspectors of fair trading found that a wholesaler of golfing equipment was swindling his retailers by including one box of substandard golf balls to every nine boxes of top grade balls he sold them. Each box contained 6 golf balls, and the external appearance of all the balls was identical. However, the substandard balls were each 1 gram too light. The retailers were informed of this discrepancy. The boxes all arrived in packs of ten, each with one substandard box — but which one?

Phoebe Fivewood, the professional at a prestigious golf course, had just taken delivery of a large order and needed to identify the defective ones quickly. She soon found a way to do this using a pair of scales (not pan balances) which required only one weighing on each scale for each batch of ten boxes. How did she do it? Note that she did not need to know what a golf ball should weigh.

Problem 6. The telephone numbers in a town run from 00000 to 99999. A common error in dialing on a standard keypad is to punch in a digit **vertically** adjacent to the intended one. So, on a standard dialing keypad, 6 could be erroneously entered as 3 or 9 (but not as 2, 5 or 8).



Assuming that no other kinds of errors are made, how should a sixth digit be added to each telephone number so that no wrong numbers will be reached because of a dialing error?

Problem 7. There are four vials labeled A, B, C and D. Exactly two vials contain a deadly virus and cannot be opened. The other two vials contain vaccines.
There is a device that can test exactly one of the vials, and it will flash a red light if the vial contains the virus. The device can be used eleven times. It is not entirely reliable, and it may give false results up to two times out of the eleven.

Professor Scarlet devises a method to find the vials containing the virus: Test each of vials A, B and C three times and vial D twice and record the results. Represent these results as strings of 0's and 1's using 0 for a red light and 1 for no light. This produces six code words which are eleven bits long.

(a) List the code words and the Hamming distance between each pair of code words.

(b) When the test was run the results were as follows:

$$\text{vial } A: \quad 1 \quad 1 \quad 0 \qquad \text{vial } B: \quad 0 \quad 0 \quad 0 \qquad \text{vial } C: \quad 0 \quad 0 \quad 1 \qquad \text{vial } D: \quad 1 \quad 1$$

Which two vials contain the virus?

(c) Dr. Ecco claims that there is no need to test vial D, you only need to test each of the others three times. Is he correct?

Problem 8. Does there exist a single-error-correcting code with three code words each of which consists of three bits? Explain your answer.