MATH 214 (R1) Winter 2008
Intermediate Calculus I

Problem Set #9

Completion Date: Friday March 28, 2008

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Question 1. **[Sec. 14.2, # 8]** Given the vector equation

$$\mathbf{r}(t) = 2\sin t\,\mathbf{i} + 3\cos t\,\mathbf{j},$$

(a) sketch the plane curve with the given vector equation,

(b) find $\mathbf{r}'(t)$,

(c) sketch the position vector $\mathbf{r}(t)$ and the tangent vector $\mathbf{r}'(t)$ for the value $t = \pi/3$.

Question 2. **[Sec. 14.2, # 26]** Find parametric equations for the tangent line to the curve whose parametric equations are

$$x = \ln t, \quad y = 2\sqrt{t}, \quad z = t^2, \quad 0 < t < \infty$$

at the point $(0, 2, 1)$.

Question 3. **[Sec. 14.2, # 30]** Given the curve

$$\mathbf{r}(t) = \langle \sin \pi t, 2\sin \pi t, \cos \pi t \rangle,$$

(a) Find the point of intersection of the tangent lines to the curve at the points where $t = 0$ and $t = 0.5$.

(b) Illustrate by graphing the curve and both tangent lines.

Question 4. **[Sec. 14.2, # 40]** Find $\mathbf{r}(t)$ if

$$\mathbf{r}'(t) = \sin t\,\mathbf{i} - \cos t\,\mathbf{j} + 2t\,\mathbf{k} \qquad \text{and} \qquad \mathbf{r}(0) = \mathbf{i} + \mathbf{j} + 2\,\mathbf{k}.$$

Question 5. **[Sec. 14.3, # 2]** Find the length of the curve

$$\mathbf{r}(t) = \langle t^2, \sin t - t\cos t, cost + t\sin t \rangle, \quad 0 \le t \le \pi.$$

Question 6. **[Sec. 14.3, # 14]** Given the curve

$$\mathbf{r}(t) = \langle t^2, \sin t - t\cos t, cost + t\sin t \rangle, \quad t > 0$$

(a) Find the unit tangent and unit normal vectors $\mathbf{T}(t)$ and $\mathbf{N}(t)$.

(b) Use the formula

$$\kappa(t) = \frac{|\mathbf{T}'(t)|}{|\mathbf{r}'(t)|}$$

to find the curvature.

Question 7. **[Sec. 14.3, # 18]** Given the curve

$$\mathbf{r}(t) = t\,\mathbf{i} + t\,\mathbf{j} + (1+t^2)\,\mathbf{k}$$

use the formula

$$\kappa(t) = \frac{|\mathbf{r}'(t) \times \mathbf{r}''(t)|}{|\mathbf{r}'(t)|^3}$$

to find the curvature.

Question 8. **[Sec. 14.3, # 26]** Given the curve $y = \ln x$, at what point does the curve have maximum curvature? What happens to the curvature as $x \to \infty$?

Question 9. **[Sec. 14.3, # 42]** Find the equations of the normal plane and the osculating plane of the curve

$$x = t, \qquad y = t^2, \qquad z = t^3$$

at the point $(1, 1, 1)$.

Question 10. **[Sec. 15.1, # 16]** Find and sketch the domain of the function

$$f(x, y) = \sqrt{y - x}\,\ln(y + x).$$

Question 11. **[Sec. 15.1, # 18]** Find and sketch the domain of the function

$$f(x, y) = \sqrt{x^2 + y^2 - 1} + \ln(4 - x^2 - y^2).$$

Question 12. **[Sec. 15.1, # 26]** Sketch the graph of the function

$$f(x, y) = 3 - x^2 - y^2.$$

Question 13. **[Sec. 15.1, # 38]** Draw a contour map of the function

$$f(x, y) = x^2 - y^2$$

showing several level curves.