MATH 214 (R1) Winter 2008
Intermediate Calculus I

Problem Set #7

Completion Date: Friday March 7, 2008

Department of Mathematical and Statistical Sciences
University of Alberta

Question 1. [Sec. 13.1, # 8] Find the lengths of the sides of the triangle with vertices $A(1, 2, -3)$, $B(3, 4, -2)$, and $C(3, -2, 1)$. Is $\triangle ABC$ a right triangle? Is it an isosceles triangle?

Question 2. [Sec. 13.1, # 18] Show that the equation

$$4x^2 + 4y^2 + 4z^2 - 8x + 16y = 1$$

represents a sphere, and find its center and radius.

Question 3. [Sec. 13.1, # 20] Find an equation of a sphere if one of its diameters has enpoints $(2, 1, 4)$ and $(4, 3, 10)$.

Question 4. [Sec. 13.1, # 32] Describe in words the region of $\mathbb{R}^3$ represented by the equation $x^2 + y^2 = 1$.

Question 5. [Sec. 13.2, # 12] Find a vector $\mathbf{a}$ with representation given by the directed line seqment $\overrightarrow{AB}$ from $A(4, 0, -2)$ to $B(4, 2, 1)$. Draw $\overrightarrow{AB}$ and the equivalent representation starting at the origin.

Question 6. [Sec. 13.2, # 22] Let $\mathbf{a} = 3\mathbf{i} - 2\mathbf{k}$ and $\mathbf{b} = \mathbf{i} - \mathbf{j} + \mathbf{k}$. Find $|\mathbf{a}|$, $\mathbf{a} + \mathbf{b}$, $\mathbf{a} - \mathbf{b}$, $2\mathbf{a}$, and $3\mathbf{a} + 4\mathbf{b}$.

Question 7. [Sec. 13.2, # 26] Find a vector that has the same direction as $\langle -2, 4, 2 \rangle$ but has length 6.

Question 8. [Sec. 13.3, # 18] Find the angle between the vectors

$$\mathbf{a} = \langle 6, -3, 2 \rangle \qquad \text{and} \qquad \mathbf{b} = \langle 2, 1, -2 \rangle.$$

(First find an exact expression and then approximate to the nearest degree.)

Question 9. [Sec. 13.3, # 24] Determine whether the given vectors are orthogonal, parallel, or neither.

(a) $\mathbf{u} = \langle -3, 9, 6 \rangle$, $\quad \mathbf{v} = \langle 4, -12, -8 \rangle$

(b) $\mathbf{u} = \mathbf{i} - \mathbf{j} + 2\mathbf{k}$, $\quad \mathbf{v} = 2\mathbf{i} - \mathbf{j} + \mathbf{k}$

(c) $\mathbf{u} = \langle a, b, c \rangle$, $\quad \mathbf{v} = \langle -b, a, 0 \rangle$

Question 10. [Sec. 13.3, # 26] For what values of b are the vectors $\langle -6, b, 2 \rangle$ and $\langle b, b^2, b \rangle$ orthogonal?

Question 11. [Sec. 13.3, # 38] Find the scalar and vector projections of $\mathbf{b}$ onto $\mathbf{a}$ if

$$\mathbf{a} = \langle -1, -2, 2 \rangle \qquad \text{and} \qquad \mathbf{b} = \langle 3, 3, 4 \rangle.$$