MATH 214 (R1) Winter 2008
Intermediate Calculus I

Problem Set #4

Completion Date: Monday February 11, 2008

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Question 1. **[Sec. 12.9, # 6]** Find a power series representation for the function

$$f(x) = \frac{1}{1+9x^2}$$

and determine the interval of convergence.

Question 2 **[Sec. 12.9, # 16]** Find a power series representation for the function

$$f(x) = \frac{x^2}{(1-2x)^2}$$

and determine the radius of convergence.

Question 3. **[Sec. 12.9, # 18]** Find a power series representation for the function

$$f(x) = \arctan(x/3)$$

and determine the radius of convergence.

Question 4. **[Sec. 12.9, # 30]** Use a power series representation to approximate the integral

$$f(x) = \int_0^{1/2} \frac{dx}{1+x^6}$$

to six decimal places.

Question 5. **[Sec. 12.9, # 32]** Show that the function

$$f(x) = \sum_{n=0}^{\infty} \frac{(-1)^n\, x^{2n}}{(2n)!}$$

is a solution to the differential equation

$$f''(x) + f(x) = 0.$$

Question 6. **[Sec. 12.10, # 4]** Find the Maclaurin series for

$$f(x) = \sin 2x$$

using the definition of a Maclaurin series. [Assume that f has a power series expansion. Do not show that $R_n(x) \to 0$.] Also find the associated radius of convergence.

Question 7. **[Sec. 12.10, # 14]** Find the Taylor series for

$$f(x) = \ln x$$

centered at the value $a = 2$. [Assume that f has a power series expansion. Do not show that $R_n(x) \to 0$.]

Question 8. **[Sec. 12.10, # 22]** Prove that the Maclaurin series for

$$f(x) = \cosh x$$

represents $\cosh x$ for all x.

Question 9. **[Sec. 12.10, # 30]** Use a known Maclaurin series to obtain the Maclaurin series for the function

$$f(x) = \cos^2 x.$$

Hint: Use $\cos^2 x = \dfrac{1}{2}(1 + \cos 2x)$.

Question 10. **[Sec. 12.10, # 46]** Use series to approximate the definite integral

$$\int_0^{1/2} x^2 e^{-x^2}\, dx$$

to within the accuracy $|\text{error}| < 0.001$.

Question 11. **[Sec. 12.10, # 56]** Find the sum of the series

$$\sum_{n=0}^{\infty} \frac{(-1)^n \pi^{2n}}{6^{2n}(2n)!}.$$

Question 12. **[Sec. 12.12, # 16a,b]** Approximate

$$f(x) = \cos x$$

by a Taylor polynomial T_n with degree $n = 4$ at the number $a = \dfrac{\pi}{3}$, and use Taylor's Inequality to estimate the accuracy of the approximation $f(x) \approx T_n(x)$ when x lies in the interval $0 \le x \le 2\pi/3$.

Question 13. **[Sec. 12.12, # 20a,b]** Approximate

$$f(x) = x \ln x$$

by a Taylor polynomial T_n with degree $n = 3$ at the number $a = 1$, and use Taylor's Inequality to estimate the accuracy of the approximation $f(x) \approx T_n(x)$ when x lies in the interval $1/2 \le x \le 3/2$.

Question 14. **[Sec. 12.12, # 26]** How many terms of the Maclaurin series for $\ln(1 + x)$ do you need to use to estimate $\ln 1.4$ to within 0.001 ?