MATH 214 (R1) Winter 2008
Intermediate Calculus I

Problem Set #10

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Department of Mathematical and Statistical Sciences
University of Alberta

Question 1. **[Sec. 15.2, # 8]** Find the limit

$$\lim_{(x,y)\to(0,0)} \frac{x^2+\sin^2 y}{2x^2+y^2},$$

if it exists, or show that the limit does not exist.

Question 2. **[Sec. 15.2, # 16]** Find the limit

$$\lim_{(x,y)\to(0,0)} \frac{x\,y^4}{x^2+y^8},$$

if it exists, or show that the limit does not exist.

Question 3. **[Sec. 15.2, # 36]** Determine the set of points at which the function

$$f(x,y) = \begin{cases} \dfrac{xy}{x^2+xy+y^2} & \text{if} \quad (x,y) \neq (0,0) \\ 0 & \text{if} \quad (x,y) = (0,0) \end{cases}$$

is continuous.

Question 4. **[Sec. 15.3, # 22]** Find the first partial derivatives of the function

$$f(x,t) = \arctan\left(x\sqrt{t}\,\right).$$

Question 5. **[Sec. 15.3, # 24]** Find the first partial derivatives of the function

$$f(x,y) = \int_y^x \cos(t^2)\,dt.$$

Question 6. **[Sec. 15.3, # 30]** Find the first partial derivatives of the function

$$u = x^{y/z}.$$

Question 7. **[Sec. 15.3, # 44]** Use implicit differentiation to find $\dfrac{\partial z}{\partial x}$ and $\dfrac{\partial z}{\partial y}$ if

$$\sin(xyz) = x + 2y + 3z.$$

Question 8. **[Sec. 15.3, # 46]** Find $\dfrac{\partial z}{\partial x}$ and $\dfrac{\partial z}{\partial y}$ for

(a) $z = f(x)g(y)$ (b) $z = f(xy)$ (c) $z = f\big(x/y\big)$.

Question 9. **[Sec. 15.3, # 60]** Given the function

$$f(r,s,t) = r\ln(rs^2t^3),$$

find the partial derivatives f_{rss} and f_{rst}.

Question 10. **[Sec. 15.4, # 6]** Find an equation of the tangent plane to the surface

$$z = e^{x^2-y^2}$$

at the point $(1,-1,1)$.

Question 11. **[Sec. 15.4, # 16]** Explain why the function

$$f(x,y) = \sin(2x+3y)$$

is differentiable at the point $(-3,2)$ and find the linearization $L(x,y)$ of the function at that point.

Question 12. **[Sec. 15.4, # 18]** Find the linear approximation to the function

$$f(x,y,z) = \ln(x-3y)$$

at the point $(7,2)$ and use it to approximate $f(6.9, 2.06)$. Illustrate by graphing f and the tangent plane.

Question 13. **[Sec. 15.4, # 34]** Use differentials to estimate the amount of metal in a closed cylindrical can that is 10 cm high and 4 cm in diameter if the metal in the top and bottom is 0.1 cm thick and the metal in the sides is 0.05 cm thick.

Question 14. **[Sec. 15.5, # 10]** Use the Chain Rule to find $\dfrac{\partial z}{\partial s}$ and $\dfrac{\partial z}{\partial t}$ if

$$z = e^{xy}\tan y, \qquad x = s+2t, \qquad y = s/t.$$

Question 15. **[Sec. 15.5, # 14]** Let

$$W(s,t) = F(u(s,t), v(s,t))$$

where F, u, and v are differentiable, and where

$$u(1,0) = 2, \quad u_s(1,0) = -2, \quad u_t(1,0) = 6$$

$$v(1,0) = 3, \quad v_s(1,0) = 5, \quad v_t(1,0) = 4$$

$$F_u(2,3) = -1, \quad F_v(2,3) = 10.$$

Find $W_s(1,0)$ and $W_t(1,0)$.

Question 16. **[Sec. 15.5, # 26]** Let $Y = w\tan^{-1}(uv)$ where

$$u = r+s, \qquad v = s+t, \qquad w = t+r.$$

Find the partial derivatives $\dfrac{\partial Y}{\partial r}$, $\dfrac{\partial Y}{\partial s}$, and $\dfrac{\partial Y}{\partial t}$ when $r=1,\ s=0,\ t=1$.

Question 17. **[Sec. 15.5, # 30]** Use the equation

$$\frac{dy}{dx} = -\frac{\dfrac{\partial F}{\partial x}}{\dfrac{\partial F}{\partial y}} = -\frac{F_x}{F_y}$$

to find $\dfrac{dy}{dx}$ if

$$\sin x + \cos y = \sin x\,\cos y.$$