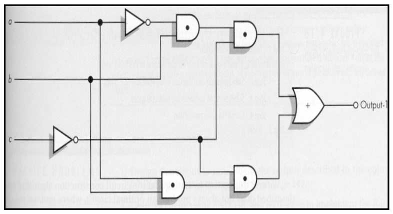
Building Computer Circuits

Chapter 4.4



Purpose

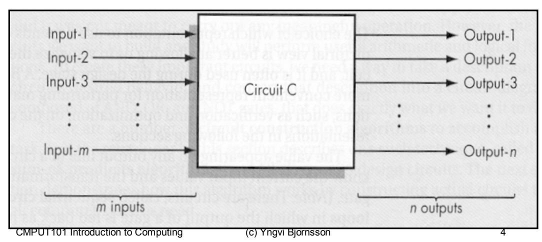
- We have looked at so far how to build logic gates from transistors.
- Next we will look at how to build circuits from logic gates, for example:
  - A circuit to check if two numbers are equal.
  - A circuit to add two numbers.
- Gates will become our new building blocks:
  - Human body: cells ➔ organs ➔ body
  - Computers: gates ➔ circuits ➔ computer

Circuit

- A <u>circuit</u> is a collection of interconnected logic gates:
  - that transforms a set of binary inputs into a set of binary outputs, and
  - where the values of the outputs depend only on the current values of the inputs
- These kind of circuits are more accurately called <u>combinatorial circuits</u>.

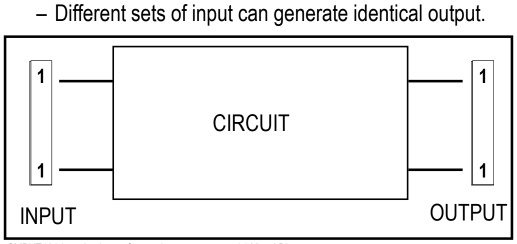
Circuit (external view)

- A circuit can have any number of inputs and outputs:
  - Number of inputs and outputs can differ.
  - The inputs and outputs are either 0 or 1.



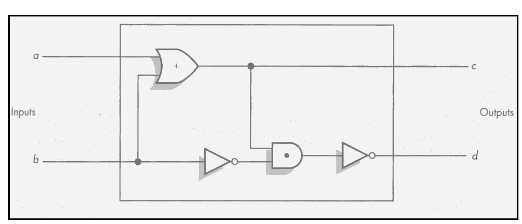
Circuit (external view cont.)

- Output depends only on current input values
  - Each set of input always generates the same output.
  - Different sets of input can generate identical output.



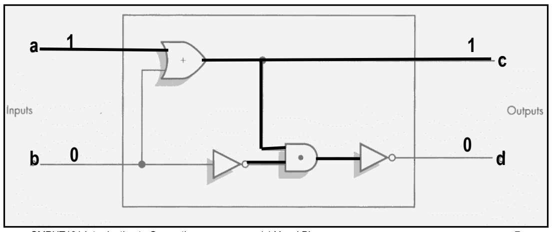
Circuit (internal view)

- Circuits are build from interconnected AND, OR and NOT gates, in a way such that each input combination produces the desired output.



Example

- What are the output values *c* and *d* given input values *a*=1, *b*=0?

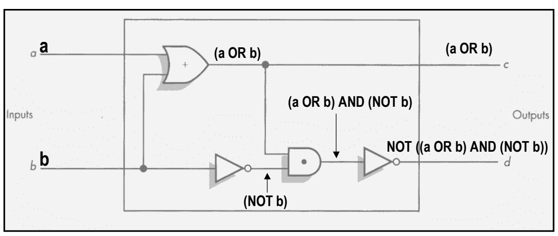


Circuit Diagrams and Boolean Expressions

- The diagrams we were looking at are called circuit diagrams.
- Relationship between circuit diagrams and Boolean expr.:
  - Every Boolean expression can be represented pictorially as a circuit.
  - Every output in a circuit diagram can be written as a Boolean expression.
- Example (output values c and d from previous diagram):
  - c = (a OR b)
  - d = NOT ((a OR b) AND (NOT b))

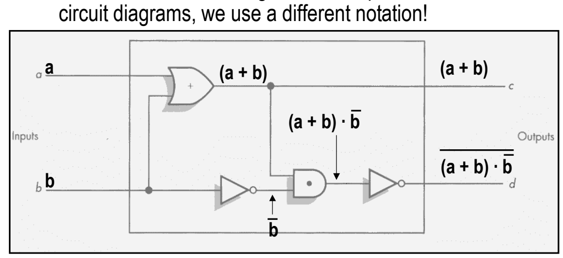
Circuits Diagram and Boolean Expressions

- Deriving Boolean expressions for the output.



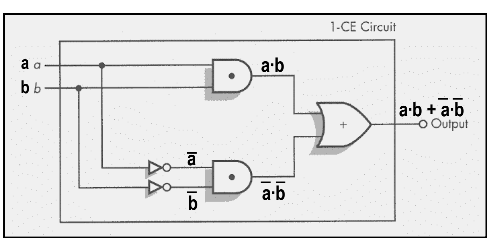
Circuits Diagram and Boolean Expressions

- Remember, when writing Boolean expressions for circuit diagrams, we use a different notation!



Example

- What Boolean expression describes the output?



Constructing Circuits

- How do we design and construct circuits?
  - We first have to know what we want the circuit to do!
  - This implies, that for all possible input combinations we must decide what the output should be.
- Once we know that, there exists methods we can use to design the layout of the circuit.
  - We will look at one such method called, sum-of-products algorithm.

Sum-of-Products Algorithm

Step 1: Truth Table Construction

Repeat steps 2, 3 and 4 for each output column

- Step 2: Sub-expression construction using AND and NOT gates
- Step 3: Sub-expression combination using OR gates
- Step 4: Circuit Diagram Production

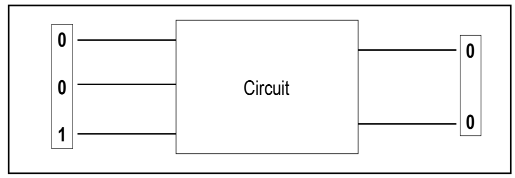
Step 5: Combine Circuit Diagrams

Step 6: Optimize Circuit (optional)

Step 7: Stop

Step 1: Truth Table Construction

- Decide what the circuit is supposed to do:
  - treat the circuit itself as a "black box"
  - only interested in input/output signals



Step 1 (cont.)

- Write the desired output for all possible input combinations:

3 inputs ➔ $2^3 = 8$ possibilities

Inputs			Outputs	
a	b	c	1	2
0	0	0	0	1
0	0	1	0	0
0	1	0	1	1
0	1	1	0	1
1	0	0	0	0
1	0	1	0	0
1	1	0	1	1
1	1	1	0	0

Step 2: Sub-expression Construction

- For each output (separately):
  - Use AND and NOT gates to construct a sub-expression for rows where the output is 1

Inputs			Outputs		
a	b	c	1	2	
0	0	0	0	1	
0	0	1	0	0	
0	1	0	1	1	← Case 1
0	1	1	0	1	
1	0	0	0	0	
1	0	1	0	0	
1	1	0	1	1	← Case 2
1	1	1	0	0	

Step 2 (cont.)

- Look at the inputs, if the value is
  - 1 then use input as is in sub-expression, (e.g. *b*)
  - 0 then use input value <u>complemented</u> (e.g. $\overline{a}$)

a	b	c	1	
…	…	…	…	
0	1	0	1	→ $\overline{a} \bullet b \bullet \overline{c}$
…	…	…	…	
1	1	0	1	→ $a \bullet b \bullet \overline{c}$
…	…	…	…	

- Why do it this way?

Each expression will evaluate to 1 for given input combination (row), but 0 for all other inputs!

Step 3: Sub-expression Combination

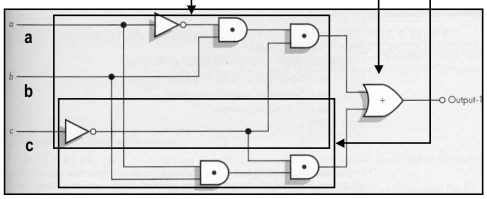
- Use OR gates to combine the sub-expressions from previous step into one expression

$$(\overline{a} \bullet b \bullet \overline{c}) + (a \bullet b \bullet \overline{c})$$

- This expression will evaluate to 1 for all input combinations that have 1 as output, but 0 for all the other input combinations (rows)!

Step 4: Circuit Diagram Production

- Construct a circuit diagram from the expression generated in previous step: $(\overline{a} \bullet b \bullet \overline{c}) + (a \bullet b \bullet \overline{c})$



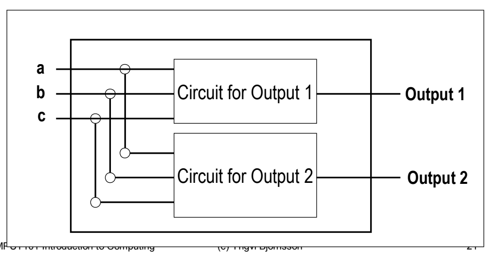
Repeat steps 2, 3, and 4 for each output

- We need to repeat steps 2, 3, 4 for each output.
- In our example, there is one more output:
  - Step2: Four sub-expressions, one for each row:
    $\overline{a} \bullet \overline{b} \bullet \overline{c}$ $\overline{a} \bullet b \bullet \overline{c}$ $\overline{a} \bullet b \bullet c$ $a \bullet b \bullet \overline{c}$
  - Step 3: Combine sub-expressions using + (OR):
    $(\overline{a} \bullet \overline{b} \bullet \overline{c}) + (\overline{a} \bullet b \bullet \overline{c}) + (\overline{a} \bullet b \bullet c) + (a \bullet b \bullet \overline{c})$
  - Step 4: Draw circuit diagram

    (see p. 694 in text-book)

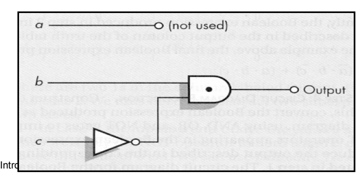
Combine Individual Circuits

- Combine the circuits for each individual output into an one larger circuit.



Optimize the Circuit

- A circuit build using this algorithm will generate the correct output, but it uses unnecessarily many gates
  - Why is that important?
  - Typically we need to optimize the circuit, by minimize the number of gates used.
- An optimized circuit for the example would look like:



Example 1: Compare-for-Equality Circuit (N-CE)

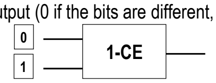
- We want to build a circuit that checks if two numbers are the same?

0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	0
=						…							=	=	=
0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	0

  - The same number if and only if <u>all</u> corresponding bits are the identical.
- First step is to build a circuit that compares two bits (can then use 16 of those to compare two 16-bit numbers!)

Ex1 --Step 1: Truth table construction

- The circuit to compare two bits has:
  - two inputs (the value of the two bits)
  - one output (0 if the bits are different, 1 if the bits are same)



- How does the truth-table look like?

Inputs		Output
a	b	
0	0	1
0	1	0
1	0	0
1	1	1

Example 1: Step 2 Construct sub-expressions

- Construct a Boolean expression for each row in the table where the output is one:

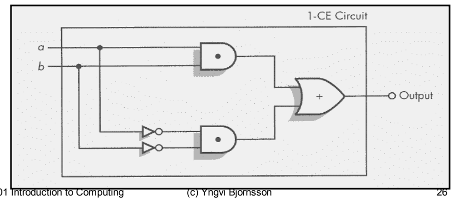
Inputs		Output	
a	b		
0	0	1	= $\overline{a}\bullet\overline{b}$
0	1	0	
1	0	0	
1	1	1	= $a\bullet b$

Example 1: Step 3 and 4

- Combine into one sub-expression using OR (+)

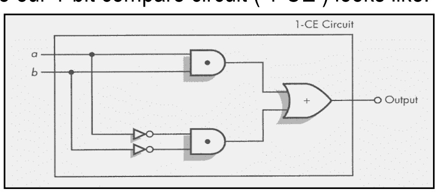
$$(\overline{a}\bullet\overline{b}) + (a\bullet b)$$

- Draw a circuit diagram



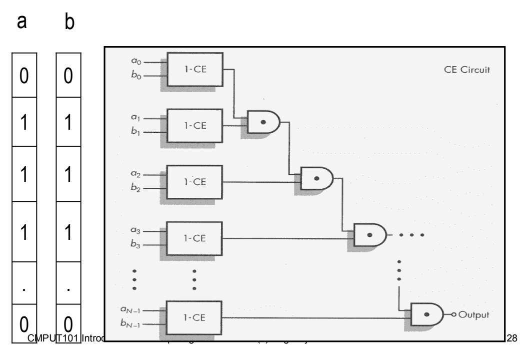
Repeat for each output

- Need to repeat step 2, 3, 4 for all outputs:
  - There is only one output, so we are done!
- So our 1-bit compare circuit (1-CE) looks like:



- But we want to compare N-bit sized numbers?

N-bit compare



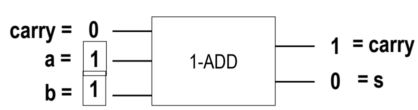
Example 2: An Addition Circuit (N-add)

- We want to build a circuit that adds two integers.
- How do we add two binary numbers
  - the same way as decimal numbers (but different base)

```
     1 1   1
    0 1 1 0 1 0 1 0   a
+   0 1 1 0 1 1 0 0   b
    1 1 0 1 0 1 1 0   s
```


Example 2: 1-ADD

- Let's start by building a circuit that adds three bits (two bits + carry)
- We can then use N of these 1-ADD circuits to add any two N-bit integers.



Ex2--Step 1: Truth table construction

Inputs			Outputs	
a	b	c	s	c
0	0	0	0	0
0	0	1	1	0
0	1	0	1	0
0	1	1	0	1
1	0	0	1	0
1	0	1	0	1
1	1	0	0	1
1	1	1	1	1

Example 2: Step 2-3 (output 1)

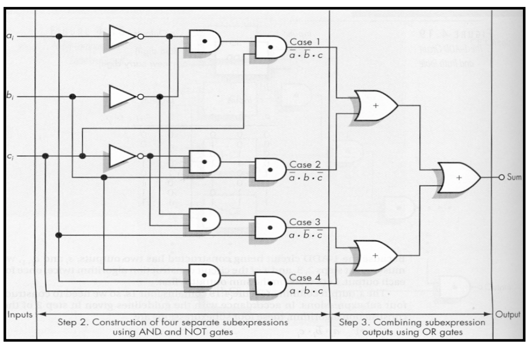
- Construct a Boolean expression for each 1-row

Inputs			Outputs	
a	b	c	s	
0	0	0	0	
0	0	1	1	→ $\overline{a} \bullet \overline{b} \bullet c$
0	1	0	1	→ $\overline{a} \bullet b \bullet \overline{c}$
0	1	1	0	
1	0	0	1	→ $a \bullet \overline{b} \bullet \overline{c}$
1	0	1	0	
1	1	0	0	
1	1	1	1	→ $a \bullet b \bullet c$

- Combine into one Boolean expression

$$s = (\overline{a} \bullet \overline{b} \bullet c) + (\overline{a} \bullet b \bullet \overline{c}) + (a \bullet \overline{b} \bullet \overline{c}) + (a \bullet b \bullet c)$$

Example 2: Step 4 Circuit Diagram (output 1)



Example 2: Step 2-3-4 (output 2)

- Step2 : Construct a Boolean expression for each 1-row

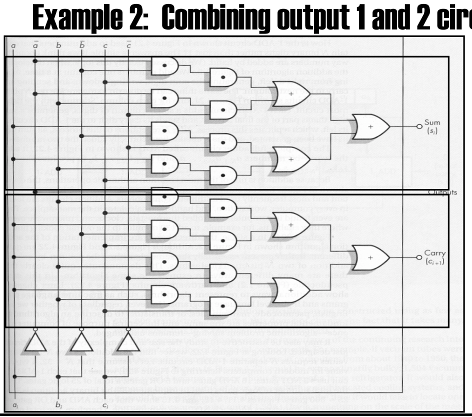
a	b	c	carry	
0	0	0	0	
0	0	1	0	
0	1	0	0	
0	1	1	1	→ $\overline{a} \bullet b \bullet c$
1	0	0	0	
1	0	1	1	→ $a \bullet \overline{b} \bullet c$
1	1	0	1	→ $a \bullet b \bullet \overline{c}$
1	1	1	1	→ $a \bullet b \bullet c$

- Step 3: Combine into one Boolean expression

$$s = (\overline{a} \bullet b \bullet c) + (a \bullet \overline{b} \bullet c) + (a \bullet b \bullet \overline{c}) + (a \bullet b \bullet c)$$

- Step 4: Draw a circuit diagram (not shown)

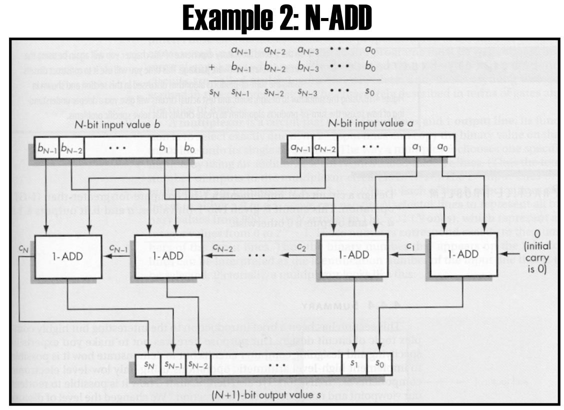
Example 2: Combining output 1 and 2 circuits



s

carry

Example 2: N-ADD



Example 2: Optimize the circuit

- Each 1-ADD circuit has 25 gates (47 transistors)
  - 16 AND gates (x 2 transistors)
  - 6 OR games (x 2 transistors)
  - 3 NOT gates (x 1 transistors)
- To add two 32-bits bits integers we need
  - 32 1-ADD circuits → 32 * 25 = 800 gates → 1504 transistors
- Optimized 32-bits addition circuit in modern computers uses: 500-600 transistors
  - We will not learn how to optimize circuits in this course

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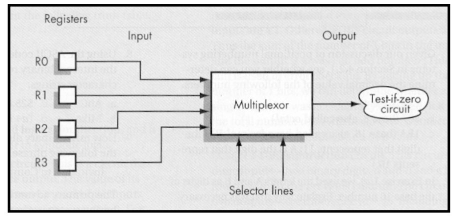
Control Circuits

Chapter 4.5



Control Circuits

- So far we have seen two types of circuits:
  - Logical (is a = b ?)
  - Arithmetic (c = a + b)
- Computers use many different logical (>, <, >=. <=, !=, …), and arithmetic (+,-,*,/) circuits.
- There are also different kind of circuits that are essential for computers ➔ control circuits
  - We will look at two different kind of control circuits, multiplexors and decoders.

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Multiplexor

- A multiplexor circuit has:
  - 2^N input lines (numbered 0, …, 2^N-1)
  - 1 output line
  - N selector lines
- The selector lines are used to choose which of the input signals becomes the output signal:
  - Selector lines interpreted as an N-bit integer
  - The signal on the input line with the corresponding number becomes the output signal.

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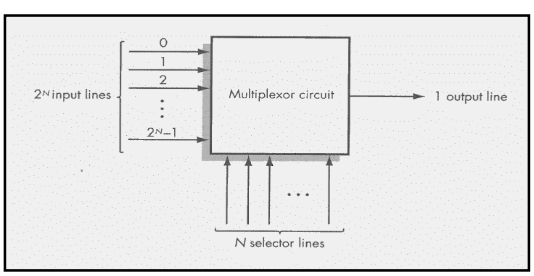
Multiplexor (cont.)



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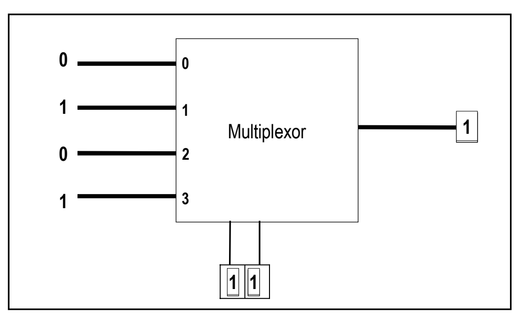
Multiplexor (cont.)



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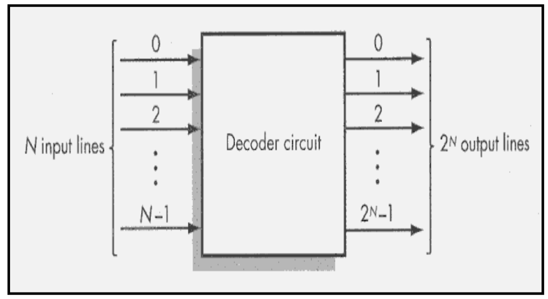
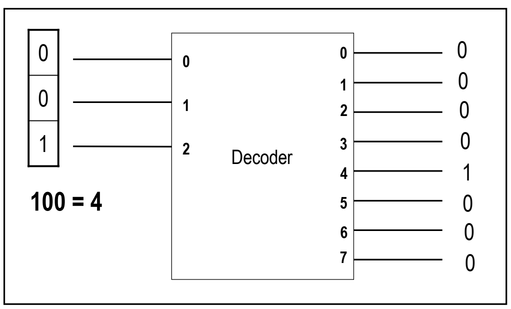
Decoder

- A decoder circuit has:
  - N input lines (numbered 0, 1, …., N-1)
  - 2^N output line (numbered 0, 1, … 2^N-1)
- Works as follows:
  - The N input lines are interpreted as a N-bit integer value.
  - The output line corresponding to the integer value is set to 1, all other to 0

Decoder (cont.)



Decoder (cont.)



Summary

- We looked at how computers represent data:
  - Internal vs External Representation
  - Basic storage unit is a binary digit ➔ <u>bit</u>
  - Data is represented internally as <u>binary data</u>.
  - Use the binary number system.
- We learned why computers use binary data:
  - Main reason is <u>reliability</u>
  - Electronic devices work best in bi-stable environment.

Summary (cont.)

- We looked at the basic building blocks used in computers:
  - Binary Storage Device = Transistor
- We saw how to build logic gates (AND, OR, NOT):
  - Transistors ➔ Gates
  - Boolean logic
- We saw how to build circuits:
  - Gates ➔ Circuits
  - Looked at logical, arithmetic, and control circuits.

Summary (cont.)

- Now that we have seen the basic building blocks (low-level view), in the next chapter we will look at the "big picture" (high-level view).
- We will look at the basic architecture underlying design of all computers:
  - Look at higher level computer components, such as processors and memory.
  - Understand better how computers execute programs.