Algorithm Discovery and Design

Chapter 2

Topics:

Representing Algorithms

Algorithmic Problem Solving

Why are Algorithms Important?

If we can discover an algorithm to perform a task, we can instruct a *computing agent* to execute it and solve the problem for us.

Representing Algorithms

- What language to use?
  - Expressive.
  - Clear, presice, and unambiguous.
- For example, we could use:
  - Natural language (e.g. English).
  - Formal programming languages (e.g. C++).
  - Something else?

Example: Adding 2 numbers

•Assume we know how to add 2 single digit numbers, but want to write an algorithm to add any 2 numbers:

$$\begin{array}{rccc} & ^{1} & & \\ & 1 & 8 & 2 \\ + & 2 & 6 & 3 \\ \hline & 4 & 4 & 5 \end{array}$$

Example using Natural Language

Initially, set the value of the variable carry to 0. When these initializations have been completed, begin looping until the value of the variable i becomes greater than m-1. First add together the values of the two digits a_i and b_i and the current value of the carry digit to get the result called c_i. Now check the value of c_i to see whether it is greater than or equal to 10. If c_i is greater than or equal to 10, then ...

Natural Languages

- English or some other natural language.
- Are **<u>not</u>** particularly good:
  - too verbose
  - unstructured
  - too rich in interpretation (ambiguous)
  - imprecise

Example using Programming Language

```
{
  int I, m, Carry;
  int a[100], b[100], c[100];
  cin >> m;
  for ( int j = 0 ; k <= m-1 ; j++ ) {
    cin >> a[j];
    cin >> b[j];
  }
  Carry = 0;
  i = 0;
  while ( i < m ) { …
```


Programming Languages

- Are **not** particularly good either:
  - Too many implementation details to worry about
  - Too rigid syntax
- Easy to lose sight of the real task
  - We don't see the forest because of all the trees!

Pseudo-code

- We need a compromise between the two:
  ➔ Pseudo-code
- Computer scientists use pseudo-code to express algorithms:
  - English like constructs (or other natural language), but
  - modeled to look like statements in typical programming languages.

Pseudo-code for the Addition Algorithm

Step	Operation
1	Get the value of $a_{m-1}, \ldots, a_0$
2	Get the value of $b_{m-1}, \ldots, b_0$
3	Set the value of *carry* to 0
4	Set the value of *i* to 0
5	Repeat steps 6-8 until *i* greater than *m-1*
6	Set the value of c_i to $a_i + b_i + carry$
7	If $c_i >= 10$, then set c_i to $c_i - 10$ and *carry* to 1; otherwise set the value of *carry* to 0
8	Set value of *i* to *i* +1 (look at next digit)
9	Set c_m to *carry*
10	Print out the final answer $c_m, c_{m-1}, \ldots c_0$
11	Stop.

What kind of operations do we need?

- Getting input and producing output
  - Get the two numbers
  - Display the outcome
- Referring to values within our algorithm
  - Add together the rightmost digits of the two numbers
  - Add together a_0 and b_0
- Doing something if some condition is true
  - If the outcome is greater or equal to 10 then ...
- Doing something repeatedly
  - Do this for all the digits in the numbers ...

Pseudo-code Primitives

Three basic kind of operations:

- Sequential
  - Computation (Set …)
  - Input/Output (Get ... / Print ...)
- Conditional
  - If … Else
  - If …
- Iterative / looping
  - Repeat ...
  - While ...

Computation

General format:

Set the value of <variable> to <expression>

Performs a computation and stores the result.

Example:

Set the value of *C to (A + B)*

Set the value of *location* to 0

Set the value of *GPA to (sum / count)*

Variables

A variable is a named storage.

- A value can be stored into it, overwriting the previous value
- Its value can be copied

Examples:

Set the value of *A* to 3

The variable A holds the value 3 after its execution

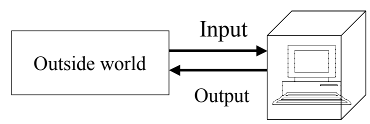
Set the value of *A* to (*A*+1)

Same as: add 1 to the value of *A* (*A* is now 4)

Not too Strict on Syntax

- Pseudo-code is kind of a programming language without a rigid syntax, for example we can write:
  - Set the value of A to (B+C)
- as
  - Set A to (B+C)
- Or even:
  - Set the value of *sum* to 0
  - Set the value of *GPA* to 0
- as
  - Set *sum* and *GPA* to 0

Sequential Operations - Input/Output



- The computing agent (computer) needs to communicate with the outside world:
  - INPUT operations allow the computing agent to receive from the outside world data values to use in subsequent computations.
  - OUTPUT operations allow the computing agent to communicate results of computations to the outside world.

Input

General format:

Get a value for <variable>

The computing agent (computer) suspends executions and waits for an input value.

Input - Examples

- Examples:
  - Get value for *grade*
  - Get values for N, M
- Can write:
  - Get value for N_1
  - ...
  - Get value for N_{100}
- as
  - Get value for $N_1,..., N_{100}$

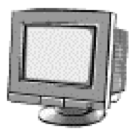
Output

General format:

Print the value of <variable>

Print the message, "<text>"

The computing agent (computer) displays the value of the variable(s).



Output - Examples

- Examples:
  - Print the value of *grade*
  - Print the message, "Hello"
- Can write:
  - Print the value of N_1
  - ...
  - Print the value of N_{100}
- as
  - Print the values of $N_1, \ldots, N_{100}$

Example

- Write an algorithm to calculate the average of three numbers.

Steps	Operations
1	Get values for *N1*, *N2*, and *N3*
2	Set the value of *Average* to (*N1+N2+N3)*/3
3	Print the value of *Average*
4	Stop

Conditional Operations

```
If <condition> then
    operations for the then-part
Else
    operations for the else-part
```

1. Evaluate <*condition*> expression to see whether it is true or false.
2. If true, then execute operations in then-part
3. Otherwise, execute operations in else-part.

Conditions, or Boolean Expressions

- A *condition* is one whose value is true or false, for example:
  - 3 > 2 is greater than (true)
  - 3 = 2 is equal to (false)
  - *A* > 2 is true if *A*'s value is greater than 2 (at the time this is executed), false otherwise.

Conditions may be compounded

E1 or E2

true if at least one of them is true; false otherwise.

E.g. 3 > 2 or 2 >3 is true

E1 and E2

true if both are true; false otherwise

E.g. 3 > 2 and 2 >3 is false

not E

true if E is false, false if E is true

Example

1. Get a value for *A*
2. If *A* = 0 then
3. Print the message, "The input is zero"
   Else
4. Print the message, "The input is not zero"

1. Get a value for *grade*
2. If *grade* < 1 or *grade* > 9 then
3. Print the message, "Invalid grade"
   Else
4. Set the value of *total* to (*grade* + *total*)

Iterative Operations - Repeat

```
Repeat steps i to j until <condition> becomes true
    step i:      operation
    step i+1:    operation
    …
    step j:      operation
```

1. Execute steps i to j
2. Evaluate <condition>
3. If condition is false, go back to 1.
4. Otherwise, continue execution from step j+1.

Example

1 Get a value for *count*
2 Repeat steps 3 to 5 until (*count* >10)
3 Set *square* to (*count* * *count*)
4 Print the values of *count* and *square*
5 Add 1 to *count*

Repeat Loops

What happens when it gets executed?

If initial value for *count* is 8, we get printout

8 64

9 81

10 100

Repeat Loops

If initial value for *count* is 11, we get printout

11 121

Why?

- Because the body is executed once before any test is done!
- If need to execute loop 0 or more times we should use While-loops.

Iterative Operation - While

```
While <condition> remains true do steps i to j
    step i:      operation
    step i+1:    operation
    …
    step j:      operation
```

1. Evaluate <condition>
2. If condition is true, execute steps i to j, then go back to 1.
3. Otherwise, if condition is false,continue execution from step j+1.

Example

1 Get a value for *count*
2 While *count* < 10 do
3 Set *square* to (*count* * *count*)
4 Print the values of *count* and *square*
5 Add 1 to *count*
6 Stop

While Loops

What happens when it gets executed?

If *count* starts with 7, we get printout

7 49
8 64
9 81

What if *count* starts with 11?

Nothing is printed, loop is executed 0 times.

Example: Multiply (via addition)

Steps	Operations
1	Get values for *N* and *M*
2	Set the value of *Result* to 0
3	While *M* > 0 do steps 4 and 5
4	Add *N* to the value of *Result*
5	Subtract 1 from *M*
6	Print the value of *Result*
7	Stop

N * M Example

Suppose initially *N* = 3 and *M* = 4. During computation, the variable *Result* held the following values, in that order:

Result: **0**
3
6
9
12

Infinite Loops

Danger:

A loop can be infinite due to non-changing conditions1

Example 1:

Repeat until 2 > 3
loop body

2 > 3 is false all the time.

Example 2:

While 3 > 2 do
loop body

3 >2 true all the time.

Why do these two algorithms not terminate?

1. Set the value of *i* to 1
2. While *i* < *10* do step 3
3. Print value of *i*
4. Stop

1. Set the value of *A* to 1
2. While *A* is an odd number do
3. Add 2 to the value of *A*
4. Print the value of *A*
5. Stop

Addition Algorithm Revisited

Step	Operation
1	Get the value of a_{m-1}, …, a_0
2	Get the value of b_{m-1}, …, b_0
3	Set the value of *carry* to 0
4	Set the value of *i* to 0
5	Repeat steps 6-8 until *i* greater than *m-1*
6	Set the value of c_i to $a_i + b_i + carry$
7	If $c_i >= 10$, then set c_i to $c_i - 10$ and *carry* to 1; otherwise set the value of *carry* to 0
8	Set value of *i* to *i* +1 (look at next digit)
9	Set c_m to *carry*
10	Print out the final answer c_m, c_{m-1}, … c_0
11	Stop.

Summary of Pseudocode

Sequential

Set the value of *variable* to *expression*

Input and Output

Get a value ……; Print …...

Conditional

If a *condition* is true then
  the first set of operations
else
  the second set of operations

Summary of Pseudocode

Iterative:

Repeat until a condition becomes true
  the loop body

While a condition remains true do
  the loop body

Exercises

I. Compute the average of 3 grades (1-9); if any one is 0 or negative, a message "Bad data" is printed

Get values for *x, y, z*
If *x* < 1 or *y* < 1 or *z* < 1 then
  Print message, "Bad data"
Else
  Set *Average* to (*x* + *y* + *z*) / 3
  Print the value of *Average*
Stop

Exercises

II. Compute the sum of *n* integers where *n* > 0

Get value for *n*, the number of integers
Get values for $I_1, I_2, …, I_n$, a list of *n* integers
Set the value of *Sum* to 0
Set the value of *k* to 1
Repeat until *k* > *n*
  Add I_k to *Sum*
  Add 1 to *k*
End of the loop
Print the value of *Sum*
Stop

Exercises

III. What does the following algorithm do?

Repeat until *A* > 0
  Print message, "Enter an integer"
  Get a value for *A*
End of the loop
Stop

IV. Write an algorithm that does the same but using a while loop instead of a repeat loop.

RECALL: Algorithms & Computing Agents

If we can discover an algorithm to perform a task, we can instruct a *computing agent* to execute it to solve the problem for us.

Algorithmic Problem Solving

Algorithm discovery

The process of finding a solution to a given problem

Typical Steps:

1. Understand the problem
2. Divide it into sub-problems
3. Sketch and refine, probably repeatedly
4. Test the correctness

Sequential search: an Example

Find the phone number of a given *Name* in an (unsorted) list of names and their phone numbers

Names	Phone numbers
N_1	T_1
N_2	T_2
…	
N_{1000}	T_{1000}

Sequential search: 1st Attempt

1. Get values for *Name*, $N_1,\ldots,N_{1000}$, $T_1,\ldots,T_{1000}$
2. If *Name* = N_1 then print the value of T_1
3. If *Name* = N_2 then print the value of T_2

…

1000. If *Name* = N_{999} then print the value of T_{999}
1001. If Name = N_{1000} then print the value of T_{1000}
1002. Stop

Sequential search: Using A Loop

Get values for *Name*, $N_1,\ldots, N_{1000}$, $T_1,\ldots, T_{1000}$
Set the value *i* to 1 and the value of *Found* to NO
Repeat until *Found* = Yes or *i > 1000*
    If *Name* = N_i then
        Print the value of T_i
        Set the value of *Found* to YES
    Else
        Add 1 to the value of *i*
End of loop
Stop

Selection: Find The Largest Number

Given a list of variables $A_1, A_2, \ldots, A_n$, find the largest value and its (first) location

Location	*A1*	*A2*	*A3*	*A4*	*A5*	*A6*	*A7*
Value	5	2	8	4	8	6	4

The largest is 8 at location 3

Idea (sketch): Go through the entire list, at each iteration find the *largest-so-far* and record its *location* *i* ↓ (at A2)

Location	A1	A2	A3	A4	A5	A6	A7
Value	5	2	8	4	8	6	4

To begin with,
set *largest-so-far* to (the value of) ***A1***
set *location* to 1
set *i* to 2

i ↓ (at A3)

Location	A1	A2	A3	A4	A5	A6	A7
Value	5	2	8	4	8	6	4

Compare ***A1*** and ***A2***
largest-so-far still holds the value of ***A1***
set *i* to *i*+1

i ↓ (at A4)

Location	A1	A2	A3	A4	A5	A6	A7
Value	5	2	8	4	8	6	4

Compare ***A1*** and ***A3***
largest-so-far now holds the value of ***A3***
location is 3
set *i* to *i*+1

i ↓ (past A7)

Location	A1	A2	A3	A4	A5	A6	A7
Value	5	2	8	4	8	6	4

Continue the similar process until *i* = 8

Selection: Find The Largest Number

Get a value for *n*, the size of the list
Get values for $A_1, A_2, \ldots, A_n$, the list to be searched
Set *largest_so_far* to A_1 and set *location* to 1
Set the value of *i* to 2
While *i* is less or equal to *n* do
    If A_i > *largest_so_far* then
        Set the value of *largest_so_far* to A_i
        Set the value of *location* to *i*
    Add 1 to the value of *i*
End of loop
Print the values of *largest_so_far* and *location*

Algorithmic Problem Solving: Summary

Two examples of algorithmic problem solving

- Sequential search

Q: On the average, how many comparisons (of names) does the algorithm make?

- Selection

Q: Design a similar algorithm to find
-the smallest value and its first location
-the largest and all the locations holding it