Algorithm Discovery and Design

Chapter 2

Topics:

Representing Algorithms

Algorithmic Problem Solving

Why are Algorithms Important?

If we can discover an algorithm to perform a task, we can instruct a *computing agent* to execute it and solve the problem for us.

Representing Algorithms

- What language to use?
  - Expressive.
  - Clear, presice, and unambiguous.
- For example, we could use:
  - Natural language (e.g. English).
  - Formal programming languages (e.g. C++).
  - Something else?

Example: Adding 2 numbers

•Assume we know how to add 2 single digit numbers, but want to write an algorithm to add any 2 numbers:

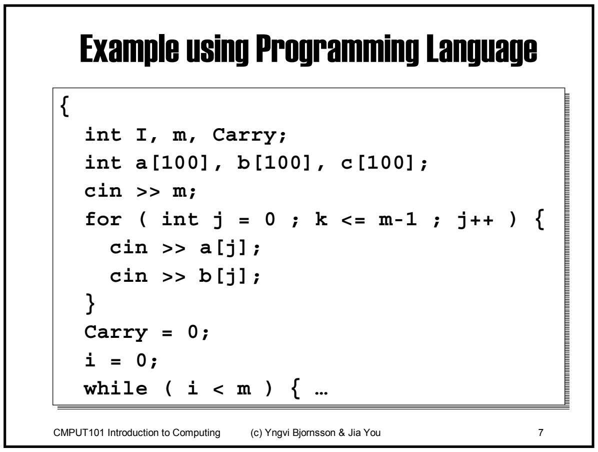
```
    1
    1 8 2
  + 2 6 3
  -------
    4 4 5
```

Example using Natural Language

> *Initially, set the value of the variable carry to 0. When these initializations have been completed, begin looping until the value of the variable i becomes greater than m-1. First add together the values of the two digits* a_i *and* b_i *and the current value of the carry digit to get the result called* c_i*. Now check the value of* c_i *to see whether it is greater than or equal to 10. If* c_i *is greater than or equal to 10, then ...*

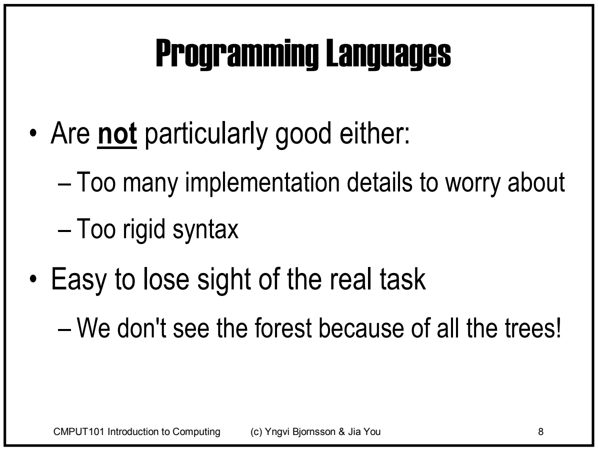
Natural Languages

- English or some other natural language.
- Are **<u>not</u>** particularly good:
  - too verbose
  - unstructured
  - too rich in interpretation (ambiguous)
  - imprecise



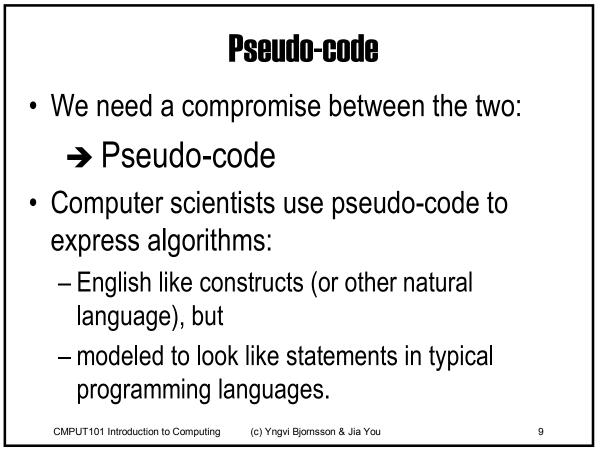
Example using Programming Language

```
{
  int I, m, Carry;
  int a[100], b[100], c[100];
  cin >> m;
  for ( int j = 0 ; k <= m-1 ; j++ ) {
    cin >> a[j];
    cin >> b[j];
  }
  Carry = 0;
  i = 0;
  while ( i < m ) { …
```



Programming Languages

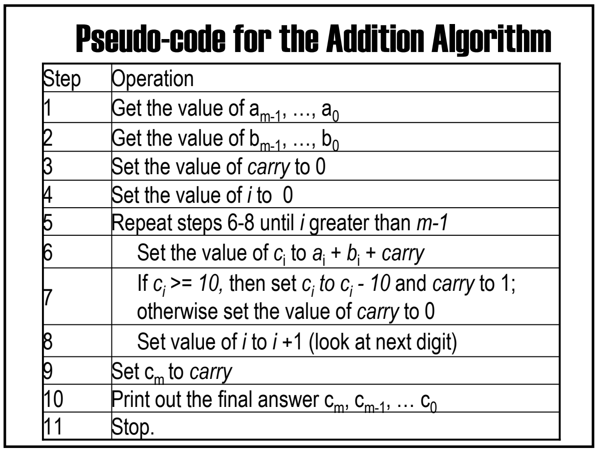
- Are **<u>not</u>** particularly good either:
  - Too many implementation details to worry about
  - Too rigid syntax
- Easy to lose sight of the real task
  - We don't see the forest because of all the trees!



Pseudo-code

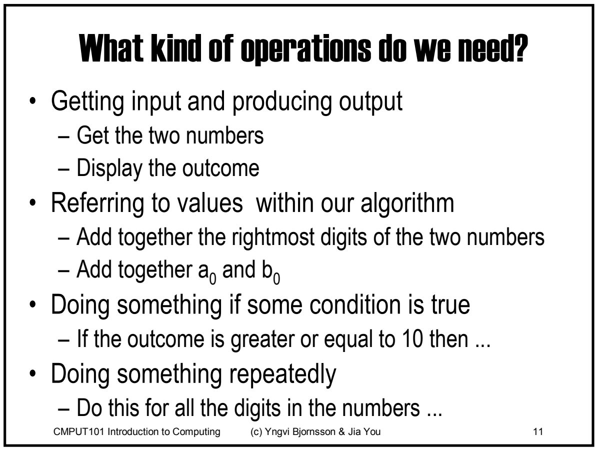
- We need a compromise between the two:

  ➔ Pseudo-code
- Computer scientists use pseudo-code to express algorithms:
  - English like constructs (or other natural language), but
  - modeled to look like statements in typical programming languages.



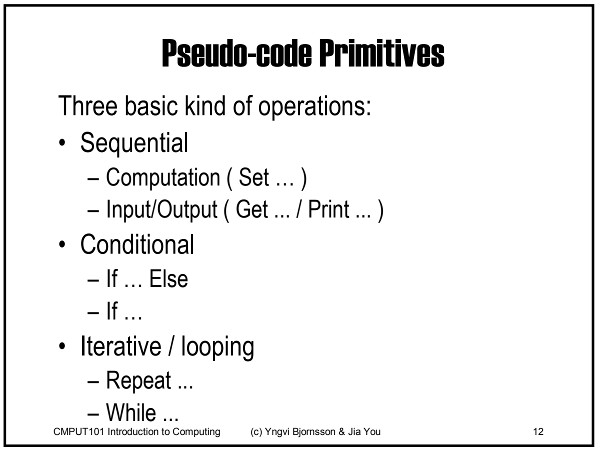
Pseudo-code for the Addition Algorithm

Step	Operation
1	Get the value of a_{m-1}, …, a_0
2	Get the value of b_{m-1}, …, b_0
3	Set the value of *carry* to 0
4	Set the value of *i* to 0
5	Repeat steps 6-8 until *i* greater than *m-1*
6	Set the value of c_i to $a_i + b_i + carry$
7	If $c_i >= 10$, then set c_i to $c_i - 10$ and *carry* to 1; otherwise set the value of *carry* to 0
8	Set value of *i* to *i* +1 (look at next digit)
9	Set c_m to *carry*
10	Print out the final answer c_m, c_{m-1}, … c_0
11	Stop.



What kind of operations do we need?

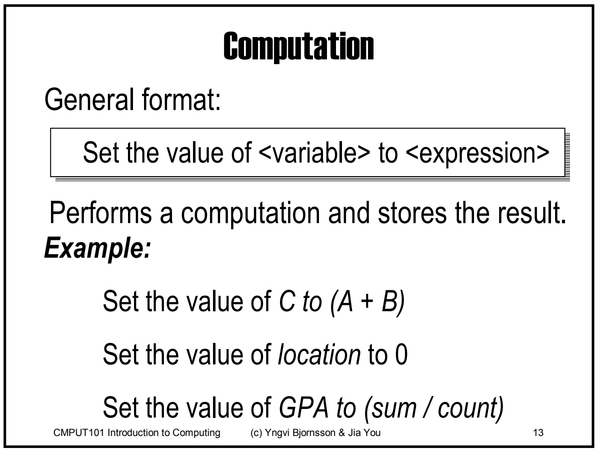
- Getting input and producing output
  - Get the two numbers
  - Display the outcome
- Referring to values within our algorithm
  - Add together the rightmost digits of the two numbers
  - Add together a_0 and b_0
- Doing something if some condition is true
  - If the outcome is greater or equal to 10 then ...
- Doing something repeatedly
  - Do this for all the digits in the numbers ...



Pseudo-code Primitives

Three basic kind of operations:

- Sequential
  - Computation (Set …)
  - Input/Output (Get ... / Print ...)
- Conditional
  - If … Else
  - If …
- Iterative / looping
  - Repeat ...
  - While ...



Computation

General format:

Set the value of <variable> to <expression>

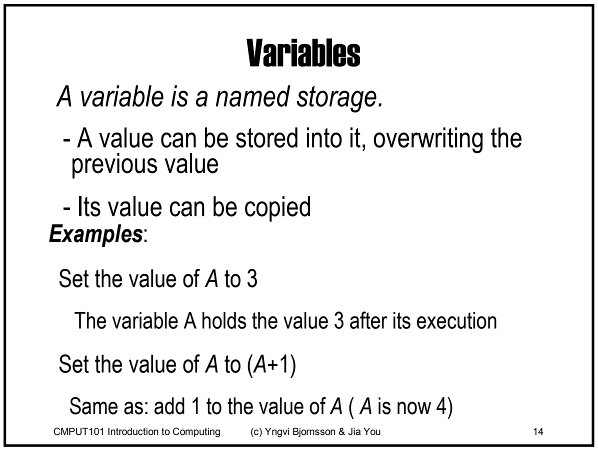
Performs a computation and stores the result.

Example:

Set the value of *C to (A + B)*

Set the value of *location* to 0

Set the value of *GPA to (sum / count)*



Variables

A variable is a named storage.

- A value can be stored into it, overwriting the previous value
- Its value can be copied

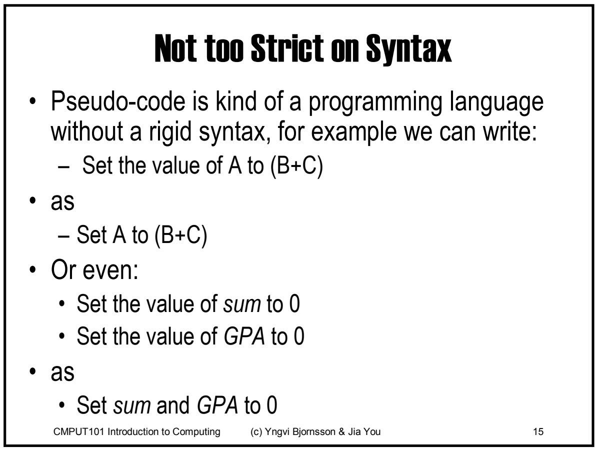
Examples:

Set the value of *A* to 3

The variable A holds the value 3 after its execution

Set the value of *A* to (*A*+1)

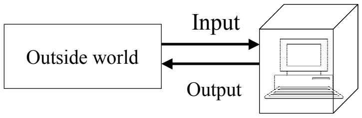
Same as: add 1 to the value of *A* (*A* is now 4)



Not too Strict on Syntax

- Pseudo-code is kind of a programming language without a rigid syntax, for example we can write:
  - Set the value of A to (B+C)
- as
  - Set A to (B+C)
- Or even:
  - Set the value of *sum* to 0
  - Set the value of *GPA* to 0
- as
  - Set *sum* and *GPA* to 0

Sequential Operations - Input/Output



- The computing agent (computer) needs to communicate with the outside world:
  - INPUT operations allow the computing agent to receive from the outside world data values to use in subsequent computations.
  - OUTPUT operations allow the computing agent to communicate results of computations to the outside world.

Input

General format:

Get a value for <variable>

The computing agent (computer) suspends executions and waits for an input value.

Input - Examples

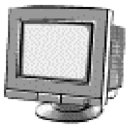
- Examples:
  - Get value for *grade*
  - Get values for N, M
- Can write:
  - Get value for N_1
  - ...
  - Get value for N_{100}
- as
  - Get value for N_1,..., N_{100}

Output

General format:

```
Print the value of <variable>
Print the message, "<text>"
```

The computing agent (computer) displays the value of the variable(s).



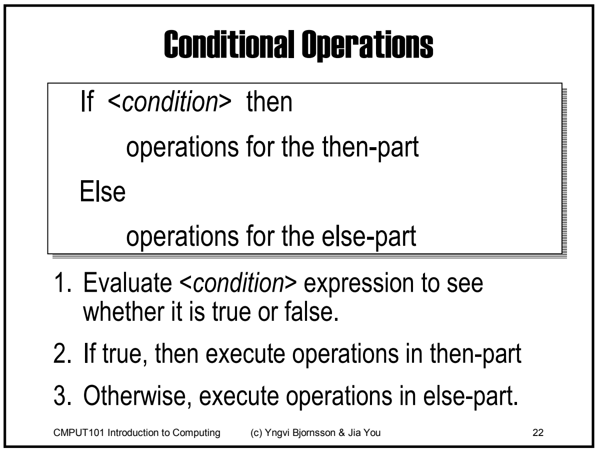
Output - Examples

- Examples:
  - Print the value of *grade*
  - Print the message, "Hello"
- Can write:
  - Print the value of N_1
  - ...
  - Print the value of N_{100}
- as
  - Print the values of $N_1, \ldots, N_{100}$

Example

- Write an algorithm to calculate the average of three numbers.

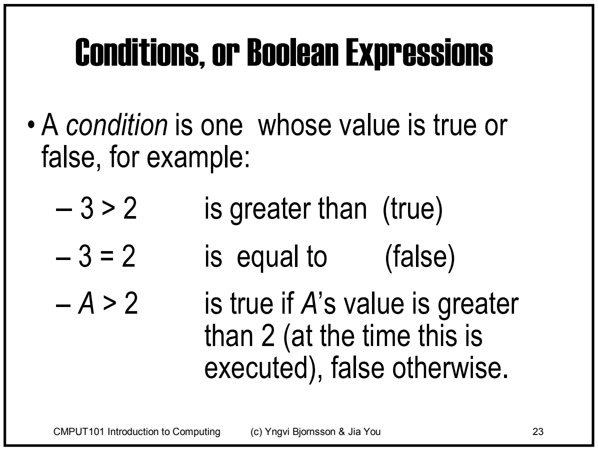
Steps	Operations
1	Get values for *N1*, *N2*, and *N3*
2	Set the value of *Average* to (*N1+N2+N3)*/3
3	Print the value of *Average*
4	Stop



Conditional Operations

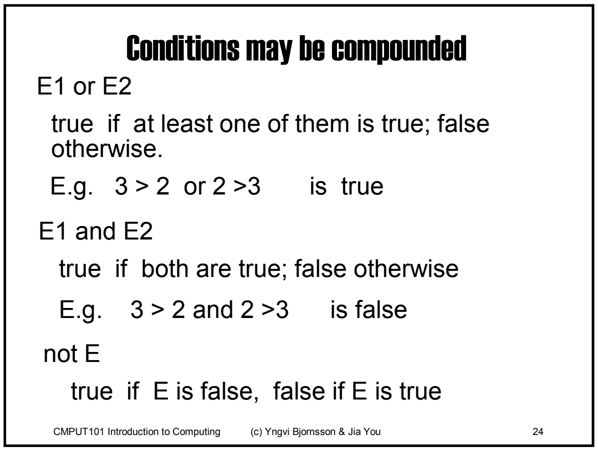
```
If <condition> then
    operations for the then-part
Else
    operations for the else-part
```

1. Evaluate <*condition*> expression to see whether it is true or false.
2. If true, then execute operations in then-part
3. Otherwise, execute operations in else-part.



Conditions, or Boolean Expressions

- A *condition* is one whose value is true or false, for example:
  - 3 > 2 is greater than (true)
  - 3 = 2 is equal to (false)
  - *A* > 2 is true if *A*'s value is greater than 2 (at the time this is executed), false otherwise.



Conditions may be compounded

E1 or E2

true if at least one of them is true; false otherwise.

E.g. 3 > 2 or 2 >3 is true

E1 and E2

true if both are true; false otherwise

E.g. 3 > 2 and 2 >3 is false

not E

true if E is false, false if E is true

Example

```
1. Get a value for A
2. If A = 0 then
3.     Print the message, "The input is zero"
   Else
4.     Print the message, "The input is not zero"
```

```
1. Get a value for grade
2. If grade < 1 or grade > 9  then
3.     Print the message, "Invalid grade"
   Else
4.     Set the value of total to (grade + total)
```

Iterative Operations - Repeat

```
Repeat steps i to j until <condition> becomes true
        step i:      operation
        step i+1:    operation
        …
        step j:      operation
```

1. Execute steps i to j
2. Evaluate <condition>
3. If condition is false, go back to 1.
4. Otherwise, continue execution from step j+1.

Example

```
1    Get a value for count
2    Repeat steps 3 to 5 until (count >10)
3        Set square to (count * count)
4        Print the values of count and square
5        Add 1 to count
```

Repeat Loops

What happens when it gets executed?

If initial value for *count* is 8, we get printout

```
8  64
9  81
10  100
```

Repeat Loops

If initial value for *count* is 11, we get printout

```
11  121
```

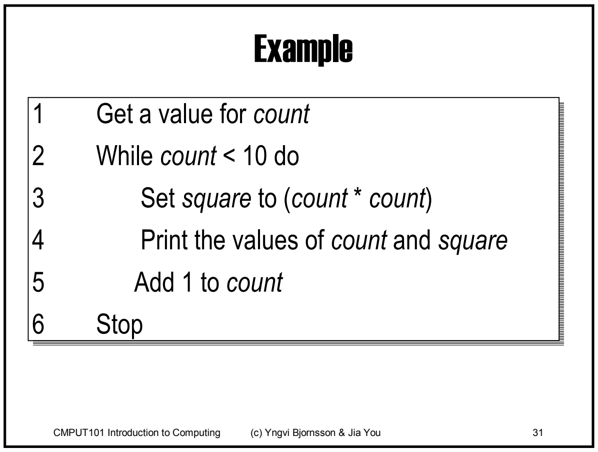
Why?

- Because the body is executed once before any test is done!
- If need to execute loop 0 or more times we should use While-loops.

Iterative Operation - While

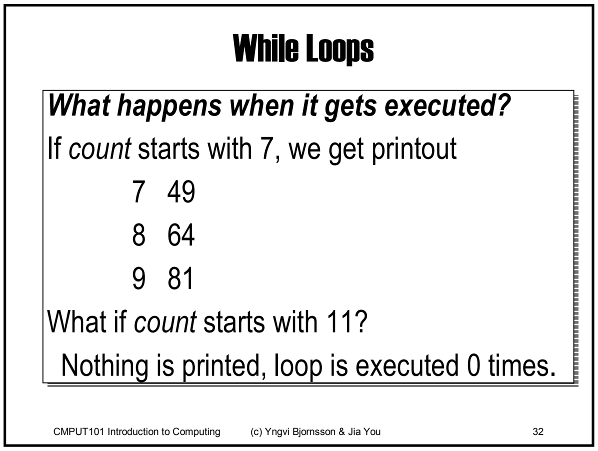
```
While <condition> remains true do steps i to j
    step i:      operation
    step i+1:    operation
    …
    step j:      operation
```

1. Evaluate <condition>
2. If condition is true, execute steps i to j, then go back to 1.
3. Otherwise, if condition is false,continue execution from step j+1.



Example

```
Get a value for count
While count < 10 do
    Set square to (count * count)
    Print the values of count and square
    Add 1 to count
Stop
```



While Loops

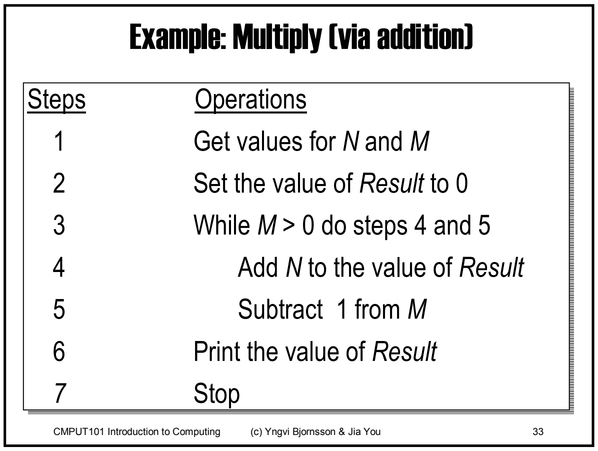
What happens when it gets executed?

If *count* starts with 7, we get printout

```
7  49
8  64
9  81
```

What if *count* starts with 11?

Nothing is printed, loop is executed 0 times.



Example: Multiply (via addition)

```
Steps     Operations
  1       Get values for N and M
  2       Set the value of Result to 0
  3       While M > 0 do steps 4 and 5
  4           Add N to the value of Result
  5           Subtract 1 from M
  6       Print the value of Result
  7       Stop
```

N * M Example

Suppose initially *N* = 3 and *M* = 4. During computation, the variable *Result* held the following values, in that order:

Result: **0**
3
6
9
12

Infinite Loops

Danger:

A loop can be infinite due to non-changing conditions1

Example 1:

```
Repeat until 2 > 3
    loop body
```

2 > 3 is false all the time.

Example 2:

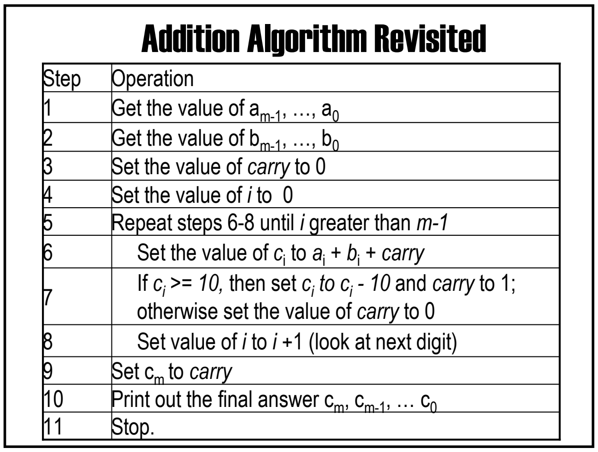
```
While 3 > 2 do
    loop body
```

3 >2 true all the time.

Why do these two algorithms not terminate?

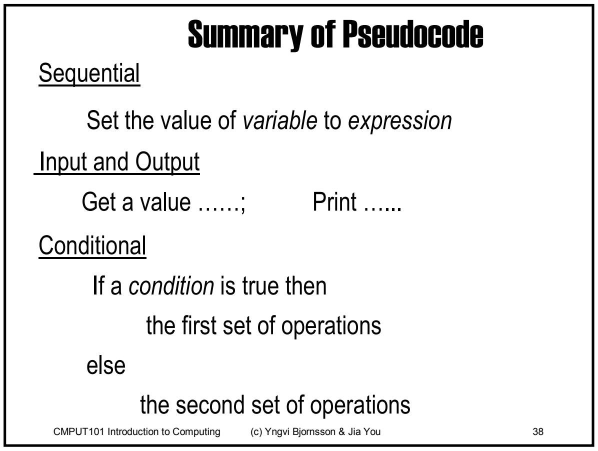
```
1.  Set the value of i to 1
2.  While i < 10 do step 3
3.          Print value of i
4.  Stop
```

```
1.  Set the value of A to 1
2.  While A is an odd number do
3.          Add 2 to the value of A
4.          Print the value of A
5.  Stop
```



Addition Algorithm Revisited

Step	Operation
1	Get the value of a_{m-1}, …, a_0
2	Get the value of b_{m-1}, …, b_0
3	Set the value of *carry* to 0
4	Set the value of *i* to 0
5	Repeat steps 6-8 until *i* greater than *m-1*
6	Set the value of c_i to $a_i + b_i + carry$
7	If $c_i >= 10$, then set c_i to $c_i - 10$ and *carry* to 1; otherwise set the value of *carry* to 0
8	Set value of *i* to *i* +1 (look at next digit)
9	Set c_m to *carry*
10	Print out the final answer c_m, c_{m-1}, … c_0
11	Stop.



Summary of Pseudocode

Sequential

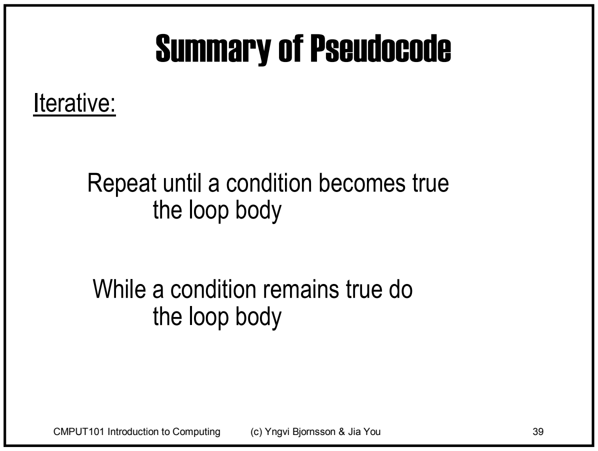
Set the value of *variable* to *expression*

Input and Output

Get a value ……; Print …...

Conditional

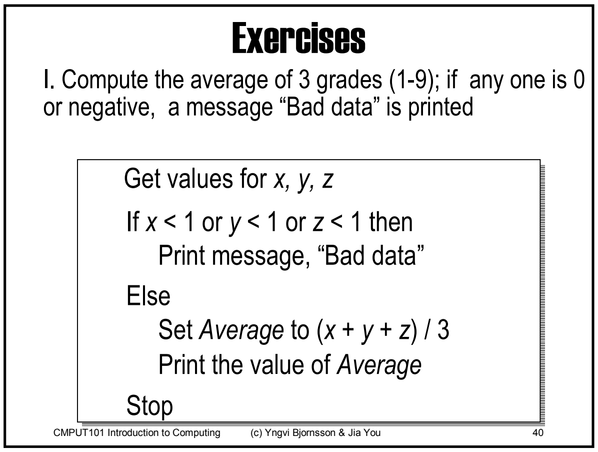
```
If a condition is true then
        the first set of operations
else
        the second set of operations
```



Summary of Pseudocode

Iterative:

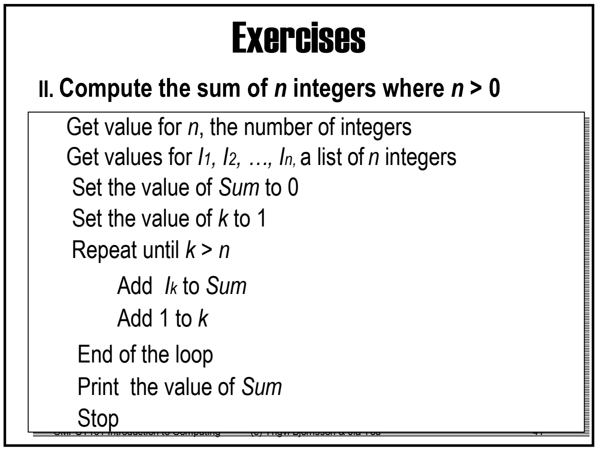
```
Repeat until a condition becomes true
        the loop body

While a condition remains true do
        the loop body
```



Exercises

I. Compute the average of 3 grades (1-9); if any one is 0 or negative, a message “Bad data” is printed

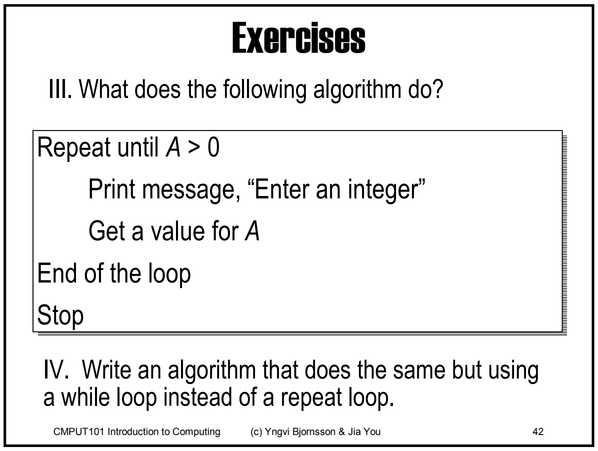
```
Get values for x, y, z
If x < 1 or y < 1 or z < 1 then
    Print message, “Bad data”
Else
    Set Average to (x + y + z) / 3
    Print the value of Average
Stop
```



Exercises

II. Compute the sum of *n* integers where *n* > 0

```
Get value for n, the number of integers
Get values for $I_1, I_2, \ldots, I_n$, a list of n integers
Set the value of Sum to 0
Set the value of k to 1
Repeat until $k > n$
    Add $I_k$ to Sum
    Add 1 to k
End of the loop
Print the value of Sum
Stop
```



Exercises

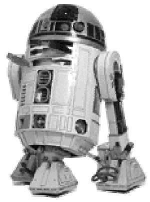
III. What does the following algorithm do?

```
Repeat until $A > 0$
    Print message, “Enter an integer”
    Get a value for A
End of the loop
Stop
```

IV. Write an algorithm that does the same but using a while loop instead of a repeat loop.

RECALL: Algorithms & Computing Agents

If we can discover an algorithm to perform a task, we can instruct a *computing agent* to execute it to solve the problem for us.



Algorithmic Problem Solving

Algorithm discovery

The process of finding a solution to a given problem

Typical Steps:

1. Understand the problem
2. Divide it into sub-problems
3. Sketch and refine, probably repeatedly
4. Test the correctness

Sequential search: an Example

Find the phone number of a given *Name* in an (unsorted) list of names and their phone numbers

Names	Phone numbers
N_1	T_1
N_2	T_2
…	
N_{1000}	T_{1000}

Sequential search: 1st Attempt

```
1.    Get values for $Name$, $N_1,\ldots,N_{1000}$, $T_1,\ldots,T_{1000}$
2.    If $Name = N_1$ then print the value of $T_1$
3.    If $Name = N_2$ then print the value of $T_2$
…
1000. If $Name = N_{999}$ then print the value of $T_{999}$
1001. If Name = $N_{1000}$ then print the value of $T_{1000}$
1002. Stop
```

Sequential search: Using A Loop

```
Get values for $Name$, $N_1,\ldots, N_{1000}$, $T_1,\ldots, T_{1000}$
Set the value $i$ to 1 and the value of Found to NO
Repeat until Found = Yes or $i > 1000$
    If $Name = N_i$ then
        Print the value of $T_i$
        Set the value of Found to YES
    Else
        Add 1 to the value of $i$
End of loop
Stop
```

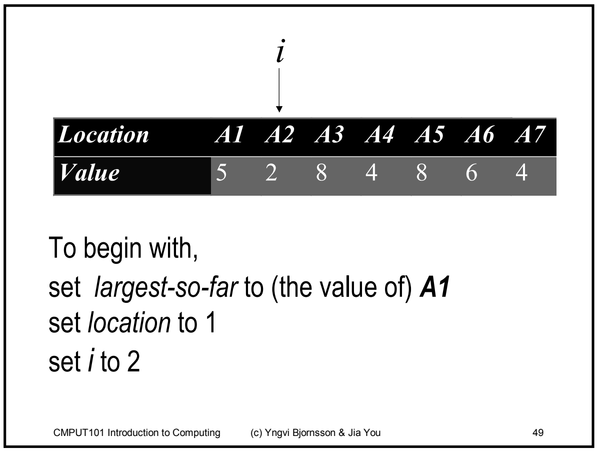
Selection: Find The Largest Number

Given a list of variables $A_1, A_2, \ldots, A_n$, find the largest value and its (first) location

Location	*A1*	*A2*	*A3*	*A4*	*A5*	*A6*	*A7*
Value	5	2	8	4	8	6	4

The largest is 8 at location 3

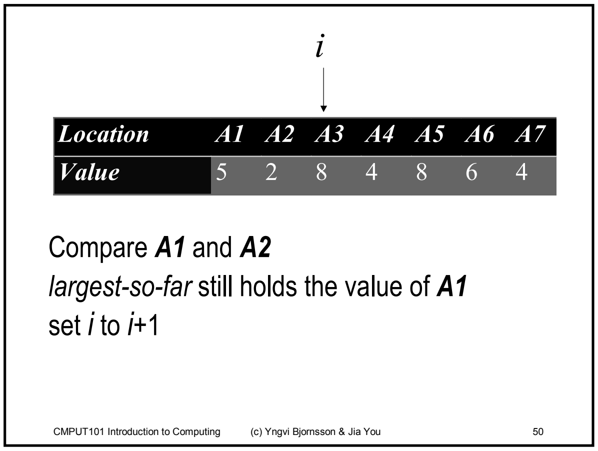
Idea (sketch): Go through the entire list, at each iteration find the *largest-so-far* and record its *location*



i

Location	*A1*	*A2*	*A3*	*A4*	*A5*	*A6*	*A7*
Value	5	2	8	4	8	6	4

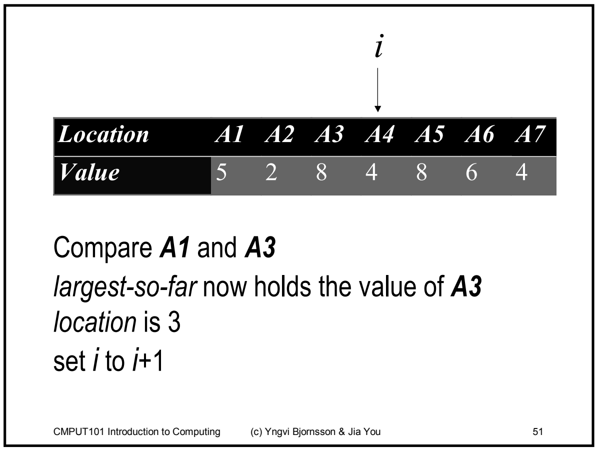
To begin with,
set *largest-so-far* to (the value of) ***A1***
set *location* to 1
set *i* to 2



i

Location	*A1*	*A2*	*A3*	*A4*	*A5*	*A6*	*A7*
Value	5	2	8	4	8	6	4

Compare ***A1*** and ***A2***
largest-so-far still holds the value of ***A1***
set *i* to *i*+1



i

Location	*A1*	*A2*	*A3*	*A4*	*A5*	*A6*	*A7*
Value	5	2	8	4	8	6	4

Compare ***A1*** and ***A3***
largest-so-far now holds the value of ***A3***
location is 3
set *i* to *i*+1

i

Location	*A1*	*A2*	*A3*	*A4*	*A5*	*A6*	*A7*
Value	5	2	8	4	8	6	4

Continue the similar process until *i* = 8

Selection: Find The Largest Number

```
Get a value for $n$, the size of the list
Get values for $A_1, A_2, \ldots, A_n$, the list to be searched
Set largest_so_far to $A_1$ and set location to 1
Set the value of $i$ to 2
While $i$ is less or equal to $n$ do
    If $A_i$ > largest_so_far then
        Set the value of largest_so_far to $A_i$
        Set the value of location to $i$
    Add 1 to the value of $i$
End of loop
Print the values of largest_so_far and location
```

Algorithmic Problem Solving: Summary

Two examples of algorithmic problem solving

- Sequential search

Q: On the average, how many comparisons (of names) does the algorithm make?

- Selection

Q: Design a similar algorithm to find
-the smallest value and its first location
-the largest and all the locations holding it