A project from professor Hao Wang (University of Alberta) for IUSEP students:

Cell division is the process by which a parent cell divides into two daughter cells, however, it takes time \( \tau \) via the cell cycle for a daughter cell to be mature for division. The cell doubling process with delay is modeled by \( 2x(t - \tau) \). The cell mortality process is modeled by \(-x(t)\), which assumes exponential delay with the per capita mortality rate one. The mathematical model is provided by the following delay differential equation:

\[
x'(t) = 2x(t - \tau) - x(t)
\]  

(1)

Questions:

1. Find the unique steady state solution and perform local stability analysis for it. Note that \( \tau \) is the only parameter which will determine stability results.

2. Write equation (1) in the form of a functional differential equation.

3. For \( \tau = 1 \) and the initial function \( x(t) = 1 \) for \(-1 \leq t \leq 0\) (starting from one cell), find the explicit solution on the interval \( 0 < t \leq 2 \) if possible.

4. For the initial function \( x(t) = 1 \) for \(-\tau \leq t \leq 0\), use DDE23 in matlab to simulate solutions for different values of the delay \( \tau \). Try your best to interpret the simulation results for the cell division process.

5. For mathematical feasibility, the model is kept as simple as possible. If you are the modeler to improve this cell division model, how do you change the model? For the improved model, please explain clearly why it is better. Answer questions 1-4 again for this new model.