The AdS/CFT Correspondence and Topological Censorship

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Abstract

In this paper we consider results on topological censorship, previously obtained by the authors in [1], in the context of the AdS/CFT correspondence. These, and further, results are used to examine the relationship of the topology of an asymptotically locally anti-de Sitter spacetime (of arbitrary dimension) to that of its conformal boundary-at-infinity (in the sense of Penrose). We also discuss the connection of these results to results in the Euclidean setting of a similar flavor obtained by Witten and Yau [2].

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1 Introduction

The AdS/CFT correspondence, first proposed by Maldacena [3], asserts the existence of a correspondence between string theory (or supergravity) on an asymptotically locally anti-de Sitter spacetime and an appropriate conformal field theory on the boundary-at-infinity. This conjectured correspondence has been supported by calculations which, for example, show a direct connection between black hole entropy as calculated classically and the number of states of the conformal field theory on the boundary-at-infinity; cf., [4] for a comprehensive review. Thus, the AdS/CFT correspondence conjecture provides new insight into the old puzzle of black hole entropy in the context of string theory. Moreover, it is believed that this conjecture, if true, may hold answers to other long-standing puzzles in gravity.

It is natural to consider what implications the topology of an asymptotically locally anti-de Sitter spacetime has for the AdS/CFT correspondence. In general, the topology of an asymptotically locally anti-de Sitter spacetime can be rather complicated. Indeed, there are well known examples that admit black holes and wormholes of various topologies [5, 6, 7, 8, 9, 10]. Furthermore one can show that there exist initial data sets with very general topology that evolve as locally anti-de Sitter spacetimes [11]. However, if the AdS/CFT correspondence conjecture is valid, one would expect the topology to be constrained in a certain manner: There should be some correspondence between the topology of an asymptotically locally anti-de Sitter spacetime and that of its boundary-at-infinity. For if the topology of an asymptotically locally anti-de Sitter spacetime were arbitrary, how could a conformal field theory that only detects the topology of its boundary-at-infinity correctly describe its physics?

Witten and Yau [2] have recently addressed this issue in the context of a generalized *Euclidean* formulation of the AdS/CFT correspondence [12]. In this formulation one considers a complete connected Riemannian manifold M^{n+1} which admits a conformal compactification (analogous to the spacetime notion of Penrose [13]), with conformal boundary (or conformal infinity) N^n. In [2], Witten and Yau show in this context that if M is an Einstein manifold of negative Ricci curvature, and if the conformal class of N admits a metric of positive scalar curvature then the nth homology of M vanishes, and, in particular, N must be connected. As discussed in [2, 14], this resolves the potential problem of the coupling (via the bulk M) of seemingly independent conformal field theories (corresponding to the components of N), in the case that the conformal class of N admits a metric of positive scalar curvature.

The results obtained by Witten and Yau [2] in the Euclidean setting do not directly address the relationship of the topology of an asymptotically locally anti-de Sitter space to that of the boundary-at-infinity in the context of spacetimes, i.e., Lorentzian manifolds, the standard arena for the AdS/CFT correspondence conjecture. Their results, however, are reminiscent of some results previously obtained by the authors [1] in the spacetime (Lorentzian) setting as a consequence of *topological censorship*. The aim of the present paper is to discuss some of these latter results in the context of the AdS/CFT correspondence, and, also, to describe the connection of these results to those of Witten and Yau. Some new results concerning the relationship of the topology of an asymptotically locally anti-de Sitter spacetime to that of the boundary-at-infinity are also presented.

Topological censorship is a basic principle of spacetime physics, which expresses the notion that the topology of the region of spacetime outside all black holes and white holes should, in some sense, be simple. According to topological censorship, in a spacetime with appropriate asymptotic structure obeying natural energy and causality conditions, any causal curve with initial and final end points on the boundary-at-infinity $\mathcal{I}$ can be continuously deformed to a curve that lies in $\mathcal{I}$ itself. Thus observers passing through the interior of such a spacetime, who remain outside all black holes and white holes, detect no topological structure not also present in the boundary-at-infinity. This result was first proved for asymptotically flat spacetimes by Friedman, Schleich and Witt [15]. More recently it has been extended by the present authors [1] to asymptotically locally anti-de Sitter spacetimes. As shown in [1], topological censorship is a powerful tool for studying the topology of the so-called *domain of outer communications* (the region outside all black holes and white holes) and the topology of black holes in asymptotically locally anti-de Sitter spacetimes in $3+1$ dimensions. Some of these results remain valid in arbitrary spacetime dimension $n+1$, $n \geq 2$, and, as discussed below, have a direct relevance to the AdS/CFT correspondence. (The proof of topological censorship, itself, is valid in arbitrary dimension $n+1$, $n \geq 2$.)

In Section 2, we introduce some basic concepts and present a statement of topological censorship for asymptotically locally anti-de Sitter spacetimes. In Section 3, we discuss the relationship of topological censorship to Witten and Yau's connectedness of the boundary result. In Section 4 we discuss how topological censorship constrains the topology of the domain of outer communications. Results both in arbitrary dimension, and in specific low dimensions of interest are considered.

2 Topological censorship in $(n+1)$-dimensional asymptotically locally anti-de Sitter spacetimes

Let M^{n+1} be an $(n+1)$-dimensional spacetime (i.e., connected time oriented Lorentzian manifold), with metric g_{ab}. Recall [18], the timelike future (resp., timelike past) of $A \subset M$, is denoted by $I^+(A, M)$ (resp., $I^-(A, M)$), and consists of all points in M that can be reached from A by a future (resp. past) directed timelike curve in M. The causal future and past of A in M, denoted $J^{\pm}(A, M)$, are defined in an analogous way using causal, rather than timelike, curves.

We use Penrose's notion of conformal infinity [13] to describe what is meant by "asymptotically locally anti-de Sitter" ("ALADS" for short). We will say that M is an ALADS spacetime provided there exists a spacetime-with-boundary M', with Lorentz metric g'_{ab}, such that (a) the boundary $\mathcal{I} = \partial M'$ is timelike, i.e., is a Lorentzian manifold in the metric induced from g'_{ab}, (b) M is the

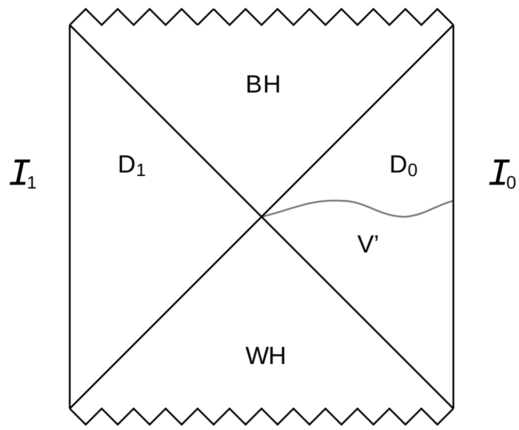

Figure 1: The Penrose diagram for Schwarzschild-AdS spacetime.

interior of M', and hence $M' = M \cup \mathcal{I}$, and (c) g_{ab} and g'_{ab} are related by: $g'_{ab} = \Omega^2 g_{ab}$, where Ω is a smooth function on M' satisfying (i) $\Omega > 0$ on M and (ii) $\Omega = 0$ and $d\Omega \neq 0$ along $\mathcal{I}$.[1] The conditions on the conformal factor Ω are standard, and guarantee that the physical metric g_{ab} falls off at a reasonable rate as one approaches the boundary-at-infinity $\mathcal{I}$. Universal anti-de Sitter spacetime (of dimension $n+1$) [4, 18] is the canonical example of an ALADS spacetime. It conformally imbeds into the Einstein static universe $\mathbb{R} \times S^n$, so that its closure M' is $\mathbb{R} \times B^n$, where B^n is a closed hemisphere of S^n, and $\mathcal{I} = \mathbb{R} \times S^{n-1}$. Another useful example to have in mind is the Schwarzschild-anti-de Sitter black hole spacetime, see Figure 1. This has a causal structure similar to extended Schwarzschild spacetime, except that the boundary-at-infinity, which consists of two components each having topology $\mathbb{R} \times S^3$, is timelike, rather than null.

To simplify our presentation a little, we shall assume in our definition of an ALADS spacetime that $\mathcal{I}$ is *spatially closed*, i.e., that each component $\mathcal{I}_\alpha$ of $\mathcal{I}$ admits a compact spacelike hypersurface S_α. From a further assumption made below, $\mathcal{I}_\alpha$ will be homeomorphic to $\mathbb{R} \times S_a$. We do not, however, make any assumptions about the topology of S_α, e.g., that it be spherical. This is what is meant by "locally" in the definition of an ALADS spacetime. Indeed many of the interesting examples in the literature have nontrivial (i.e. nonspherical) topology at infinity, see for example [9].

Topological censorship requires some form of *causal regularity*, related to the cosmic censorship hypothesis that there be no singularities visible from infinity. Recall [18], a spacetime N is said to be globally hyperbolic iff N is strongly causal (i.e., there are no closed, or "almost closed" causal curves in N), and the "causal intervals" $J^+(p,N) \cap J^-(q,N)$ are compact for all $p,q \in N$. Note that this definition still makes sense even if N is a spacetime-with-boundary; the sets $J^+(p,N) \cap J^-(q,N)$ may then meet the boundary, which is permitted. For example, the spacetime-

[1] We are using a very weak form of ALADS. Usually, it is required that the vacuum Einstein equations with negative cosmological constant hold asymptotically, in a certain prescribed sense, which then forces $\mathcal{I}$ to be timelike [16].

with-boundary $\mathbb{R} \times B^n$, where B^n is a closed hemisphere of S^n, is globally hyperbolic in this sense. Thus, although universal anti-de Sitter spacetime is not globally hyperbolic, the closure of its conformal image in the Einstein static universe, as a spacetime-with-boundary, is.

The domain of outer communications D of an ALADS spacetime M, which represents the region outside of all black holes and white holes, is the region of spacetime that can communicate with infinity, both to the future and past. In mathematical terms, $D = [I^-(\mathcal{I}, M') \cap I^+(\mathcal{I}, M')] \cap M$. Then $D' = I^-(\mathcal{I}, M') \cap I^+(\mathcal{I}, M') = D \cup \mathcal{I}$ is a spacetime-with-boundary (not necessarily connected), with timelike boundary $\mathcal{I}$. We say that an ALADS spacetime M is *causally regular* provided $D' = D \cup \mathcal{I}$ is globally hyperbolic, as a spacetime-with-boundary. The compactness of the sets $J^+(p, D') \cap J^-(q, D')$ rules out the presence of any naked singularities in D.

Finally, topological censorship requires an energy condition, such as the null energy condition (NEC): $R_{ab}X^aX^b \geq 0$ for all null vectors X^a, where R_{ab} is the Ricci tensor of spacetime. In fact, to prove topological censorship, it is sufficient to require a weaker, *averaged* version of the NEC [2], but for simplicity we will state things in terms of the NEC here. Although stated in a geometric form, the NEC can be interpreted physically by invoking the Einstein equations to relate the Ricci tensor to its sources. In particular, if the Einstein equations with cosmological constant hold, $R_{ab} - \frac{1}{2}Rg_{ab} + \Lambda g_{ab} = 8\pi T_{ab}$, then, as $g_{ab}X^aX^b = 0$ for any null vector X, we have, $R_{ab}X^aX^b = 8\pi T_{ab}X^aX^b$. Hence, the NEC depends only on the stress energy tensor, and is insensitive to the sign of the cosmological constant. In particular, the NEC is satisfied in vacuum spacetimes with negative cosmological constant.

We may now state the following version of topological censorship, which follows from Theorem 2.2 in [1].

Theorem 1 *Let M^{n+1}, $n \geq 2$, be a causally regular ALADS spacetime satisfying NEC. Let $\mathcal{I}_0$ be a connected component of the boundary-at-infinity $\mathcal{I}$. Then every causal curve in $M' = M \cup \mathcal{I}$ with initial and final end points on $\mathcal{I}_0$ is fixed end point homotopic to a curve in $\mathcal{I}_0$.*

As we describe in Section 4, Theorem 1 can be formulated in terms of fundamental groups. This enables one to study the topology (e.g., homology) of the domain of outer communications by standard algebraic topological techniques.

In the next section we describe a result closely related to Theorem 1, which can be interpreted as a Lorentzian analogue of the Witten-Yau connectedness of the boundary result.

3 Causal disconnectedness of disjoint components of the boundary-at-infinity

Theorem 1 is a consequence of the following basic result (cf., Theorem 2.1 in [1]).

[2] It is sufficient to assume the ANEC, together with the null generic condition. To avoid the null generic condition, which is not satisfied in some models, one may use a modified form of the ANEC; see [1] for further details.

Theorem 2 *Let M^{n+1}, $n \geq 2$, be a causally regular ALADS spacetime satisfying NEC. Then distinct components of the boundary-at-infinity $\mathcal{I}$ cannot communicate, i.e., if $\mathcal{I}_0$ and $\mathcal{I}_1$ are distinct components of $\mathcal{I}$ then $J^+(\mathcal{I}_0, M') \cap J^-(\mathcal{I}_1, M') = \emptyset$.*

Simply put, no causal curve can extend from one component of infinity to another. Schwarzschild-anti-de Sitter spacetime provides a clear illustration of this fact, see Figure 1. Theorem 2 is related to the fact in black hole theory [18, Prop. 9.2.8], and may be proved in a similar fashion, that so-called outer trapped surfaces cannot be seen from infinity.

Theorem 1 follows from Theorem 2 by constructing a covering space of M' in which all curves not homotopic to curves on $\mathcal{I}_0$ are unwound. Any causal curve with endpoints on $\mathcal{I}_0$ not fixed endpoint homotopic to a curve in $\mathcal{I}_0$ will begin and end on different components of the boundary-at-infinity in this covering space. But since this covering space is itself an ALADS spacetime satisfying the conditions of Theorem 2, such a curve cannot exist. Hence, Theorem 1 follows.

The implications of Theorem 2 for the AdS/CFT correspondence are immediate. In an ALADS spacetime satisfying reasonable physical conditions, any component of the boundary-at-infinity cannot communicate with any other component. Thus any field operator evaluated at a point of one component of $\mathcal{I}$ will commute with any field operator evaluated on any other component. Thus conformal field theories defined on disjoint components of the boundary-at-infinity do not interact dynamically. Clearly however, one can set up correlations in the initial vacuum states of the conformal field theories. However, any such correlations are not dynamic. It is in this sense that our Lorentzian result may be viewed as an analogue of the Euclidean connectedness of the boundary result obtained by Witten and Yau [2, 14].

Now, consider an observer in, for example, Schwarzschild-anti-de Sitter spacetime, who attempts to travel from one component of infinity to another; the observer ends up entering the black hole region. This is the typical situation: Distinct components of $\mathcal{I}$ are "screened apart" by black holes (and/or white holes). We have the following corollary.

Corollary 3 *Let M^{n+1}, $n \geq 2$, be a causally regular ALADS spacetime satisfying NEC. If $\mathcal{I}$ is not connected then M contains a black hole and/or a white hole.*

PROOF: Let $\{\mathcal{I}_\alpha\}_{\alpha \in A}$ denote the components of the boundary-at-infinity $\mathcal{I}$; hence $\mathcal{I}$ is the disjoint union of the $\mathcal{I}_\alpha$'s. Let D_α denote the domain of outer communications with respect to the component $\mathcal{I}_\alpha$, $D_\alpha = [I^-(\mathcal{I}_\alpha, M') \cap I^+(\mathcal{I}_\alpha, M')] \cap M$. It follows from Theorem 2 that the D_α's are mutually disjoint, $D_\alpha \cap D_\beta = \emptyset$, $\alpha \neq \beta$, and that the (full) domain of outer communications $D = [I^-(\mathcal{I}, M') \cap I^+(\mathcal{I}, M')] \cap M$ is the disjoint union of the D_α's.

The black hole region is the region of spacetime from which an observer cannot escape to infinity. Mathematically, it is the region $M \setminus I^-(\mathcal{I}, M')$. Time dually, the white hole region is the region $M \setminus I^+(\mathcal{I}, M')$. The union of these two regions is the region $M \setminus D$. Since spacetime is assumed to be connected, but D is not connected, D cannot be all of M, $M \setminus D \neq \emptyset$. Hence, there must be a black hole and/or a white hole in M. □

Recall that the Euclideanization procedure for transforming (via Wick rotation) a spacetime into a Riemannian manifold involves a single component of the domain of outer communications. The above proof shows that the timelike boundary-at-infinity of a single component of the domain of outer communications is connected, in formal consistency with the Witten-Yau connectedness of the boundary result.

4 The topology of the domain of outer communications

In this section we study the topology of the domain of outer communications, emphasizing the manner and extent to which it is controlled by the topology at infinity. We begin by describing, as mentioned in Section 2, how Theorem 1 can be expressed in terms of fundamental groups.

Theorem 1, when taken in conjunction with a certain covering space argument, can be shown to imply the following stronger result.

Theorem 4 *Let M^{n+1}, $n \geq 2$, be a causally regular ALADS spacetime satisfying NEC. Let $\mathcal{I}_0$ be a connected component of the boundary-at-infinity $\mathcal{I}$. Then any curve in $D_0' = D_0 \cup \mathcal{I}_0$, with initial and final end points on $\mathcal{I}_0$ is fixed end point homotopic to a curve in $\mathcal{I}_0$.*

Here, $D_0 = [I^-(\mathcal{I}_0, M') \cap I^+(\mathcal{I}_0, M')] \cap M$ is the domain of outer communications with respect to $\mathcal{I}_0$. Theorem 4 removes the qualifier in Theorem 1 that the curve in D_0' be *causal*. (Note that a causal curve with end points on $\mathcal{I}_0$ is necessarily contained D_0'.) The proof is a slight modification of the proof of Proposition 3.1 in [1].

Now, fix a point $p_0 \in \mathcal{I}_0$, and consider loops in D_0' based at p_0. Theorem 4 implies that any loop in D_0' based at p_0 can be continuously deformed to a loop in $\mathcal{I}$ based at p_0. Recalling that the inclusion map $i : \mathcal{I}_0 \hookrightarrow D_0'$ induces a natural homomorphism of fundamental groups $i_* : \Pi_1(\mathcal{I}_0) \to \Pi_1(D_0')$, the last comment can be expressed in more formal terms as follows, cf., Proposition 3.1 in [1].

Corollary 5 *Let M^{n+1}, $n \geq 2$, be a causally regular ALADS spacetime satisfying NEC. Let $\mathcal{I}_0$ be a connected component of the boundary-at-infinity $\mathcal{I}$. Then the group homomorphism $i_* : \Pi_1(\mathcal{I}_0) \to \Pi_1(D_0')$ induced by inclusion is surjective.*

Corollary 5 says that, at the fundamental group level, the topology of D_0' can be no more complicated than the topology of $\mathcal{I}_0$. In particular, if $\mathcal{I}_0$ is simply connected (e.g., $\mathcal{I}_0 \approx \mathbb{R} \times S^n$), then so is D_0', thus generalizing the result of [17]. This supports the notion of *holography*: the topology of infinity determines to some extent the topology of the bulk. Corollary 5 is a natural Lorentzian analogue of Theorem 3.3 in [2], but requires no curvature assumptions on the boundary-at-infinity.

Further information about the topology of the domain of outer communications can be obtained by considering certain spacelike hypersurfaces slicing through it. This method was used extensively in [1] to study the topology of the domain of outer communications and the topology of black holes in $3+1$ dimensional ALADS spacetimes. Some of the results obtained there extend to arbitrary dimension $n+1$, $n \geq 2$. Here we give a brief description of the basic methodology, and consider some results that hold in arbitrary dimension, as well as some results that are dimension specific. Some limitations of the methodology are also discussed.

Let M be a causally regular ALADS spacetime, and, as above, let D_0 be the domain of outer communications with respect to a component $\mathcal{I}_0$ of the boundary-at-infinity $\mathcal{I}$. Consider a spacelike hypersurface-with-boundary V' ($\dim V' = n$) in $D'_0 = D_0 \cup \mathcal{I}_0$, whose boundary Σ_∞ is an $(n-1)$-dimensional spacelike surface contained in $\mathcal{I}_0$. Using the fact that D'_0 is globally hyperbolic (as a manifold-with-boundary), V' can be chosen so that D'_0 is homeomorphic to $\mathbb{R} \times V'$, $D'_0 \approx \mathbb{R} \times V'$. (This is accomplished by extending the notion of a *Cauchy surface* [18] to spacetimes with timelike boundary, and applying a suitable analogue of [18, Prop. 6.6.8].) By restricting this homeomorphism to $\mathcal{I}$, it also follows that $\mathcal{I} \approx \mathbb{R} \times \Sigma_\infty$. Such a spacelike hypersurface V' for Schwarzschild-anti-de Sitter spacetime is depicted in Figure 1.[3]

Now, let V denote the closure of V' in the full spacetime-with-boundary $M' = M \cup \mathcal{I}$. If there are black holes present, V will meet the black hole event horizon (the boundary of the black hole region). We assume that V is a compact, orientable n-manifold-with-boundary, with interior $V' \setminus \Sigma_\infty$, and with boundary $\partial V = \Sigma_H \cup \Sigma_\infty$, where Σ_H is a compact $(n-1)$-dimensional surface contained in the event horizon. We allow Σ_H to have multiple ($k \geq 0$, say) components; each component of Σ_H corresponds to a black hole in the "time slice" V. In the Schwarzschild-AdS example depicted in Figure 1, $V \approx [0,1] \times S^{n-1}$, and Σ_H and Σ_∞ are $(n-1)$ spheres.

Since $D'_0 \approx \mathbb{R} \times V'$, the topology of D'_0 is completely determined by the topology of V', which, in turn, is completely determined by the topology of V. Moreover, the relationship between the fundamental groups of $\mathcal{I}_0$ and D'_0, as described in Corollary 5, descends to the fundamental groups of Σ_∞ and V.

Proposition 6 *Let M^{n+1}, $n \geq 2$, be a causally regular ALADS spacetime satisfying NEC. Let V be the compact spacelike hypersurface-with-boundary in $M' = M \cup \mathcal{I}$ with boundary $\partial V = \Sigma_H \cup \Sigma_\infty$, as described above. Then the homomorphism of fundamental groups $i_* : \Pi_1(\Sigma_\infty) \to \Pi_1(V)$ induced by inclusion is surjective.*

Again, this means that every loop in V can be continuously deformed to a loop in Σ_∞. Proposition 6 illustrates at the spatial level how the topology at infinity controls the topology of the bulk.

The study of the topology of the domain of outer communications with respect to the component $\mathcal{I}_0$ of the boundary-at-infinity $\mathcal{I}$ has now been reduced to the study of the topology of the

[3] In certain circumstances it is useful to modify the procedure outlined here, by constructing V' with respect to certain globally hyperbolic subregions of D'_0; cf., [1, Section IV].

spacelike slice V. We now briefly describe a few of the main results concerning the topology of V. Detailed proofs and further results may be found in [19].

Theorem 7 *Let M^{n+1}, $n \geq 2$, be a causally regular ALADS spacetime satisfying NEC, and let V be as in Proposition 6. Then the $(n-1)$-homology of V is given by, $H_{n-1}(V, \mathbb{Z}) = \mathbb{Z}^k$, where $k \geq 0$ is the number of components of Σ_H.*

Theorem 7, which was proved in $3+1$ dimensions in [1], is a consequence of Proposition 6, together with standard techniques and results in algebraic topology. Although proved in a completely different way, and not requiring a curvature condition on Σ_∞, Theorem 7 may be viewed as a spacetime analogue of Theorem 3.4 in [2]. In fact, in the absence of black holes ($k = 0$), the conclusions are formally the same.

Theorem 7 has a natural geometrical/physical intepretation. The k components of Σ_H determine k linearly independent elements of $H_{n-1}(V)$. But since $H_{n-1}(V) = \mathbb{Z}^k$, these components must span all of $H_{n-1}(V)$. Hence all of the $(n-1)$-homology of V is generated by its boundary components, and in this sense $H_{n-1}(V)$ is as simple as possible. Any topological structure in the interior of V that would generate another independent element of $H_{n-1}(V)$ cannot exist. In particular, V cannot contain any wormholes. A wormhole in V would correspond to a handle grafted to V, which would introduce an $(n-1)$-sphere that does not bound in V. This intuitive observation can be formulated in a precise way as follows.

Corollary 8 *Let M^{n+1}, $n \geq 2$, be a causally regular ALADS spacetime satisfying NEC, and let V be as in Proposition 6. Then there exists no closed n-manifold N with $b_1(N) > 0$ such that $V = U \# N$.*

In the above, # denotes the operation of connected sum, and b_1 denotes the first Betti number. In the case of a wormhole we would have $N = S^1 \times S^{n-1}$, and hence $b_1(N) > 0$. The proof of Corollary 8 involves an application of the Mayer-Vietoris sequence.

We conclude with some comments concerning results in specific dimensions. It turns out, as shown in [1], that in $3+1$ dimensions Proposition 6 is sufficient to completely determine the homology of V. Moreover, we establish in [1] a basic topological relationship between the 2-surfaces Σ_H and Σ_∞. We show that the genus of Σ_H (or, if it has more than one component, the sum of the genera of its components) is bounded above by the genus of Σ_∞. Hence the topology of the black holes is controlled by the topology at infinity.

Of special relevance to the AdS/CFT correspondence conjecture are results for ALADS spacetimes in $2+1$ and $4+1$ dimensions. In $2+1$ dimensions one has the following.

Theorem 9 *Let M^{2+1} be a causally regular ALADS spacetime satisfying NEC, and let V be as in Proposition 6. Then the 2-dimensional hypersurface V is either B^2 (a disk) or $I \times S^1$.*

Remark: The assumption made at the outset that V is orientable is not needed in Theorem 9; orientability follows from Proposition 6 and the fact that the boundary of V, being one dimensional, is necessarily orientable.

PROOF: Since, as follows from Proposition 6, $i_* : H_1(\Sigma_\infty) \to H_1(V)$ is surjective, the rank of the free part of $H_1(V)$ cannot be greater than that of $H_1(\Sigma_\infty)$, *i.e.*, $b_1(V) \leq b_1(\Sigma_\infty)$. In the case $n = 2$, Σ_∞ is a 1-manifold so $b_1(\Sigma_\infty) \leq 1$, and thus $b_1(V) \leq 1$. From the classification of 2-manifolds, V must be a closed 2-manifold minus a disjoint union of disks. The first betti number of such manifolds is $b_1 = 2g + k$ where g is the genus and $k + 1$ the number of disjoint disks; it follows that $g = 0$. Since V must have at least one boundary, the only possible topologies for V are B^2 or $I \times S^1$. □

Thus, topological censorship gives a topological rigidity theorem in $(2+1)$-dimensional gravity: V' for the domain of outer communications of each component of $\mathcal{I}$ in a $(2+1)$-dimensional black hole spacetime will have product topology.

The case of $(2 + 1)$-dimensional asymptotically flat spacetimes can be similarly treated to produce the same conclusions as Theorem 9. It follows that there are no asymptotically flat geons in $2 + 1$ dimensions.

In the case of $(4 + 1)$-dimensional spacetimes, although Proposition 6 can be used to obtain some information about the topology of V, it is not enough to fix the topology of V. This may be illustrated by considering the restricted case for which Σ_∞ is simply connected. It then follows that V is a simply connected manifold-with-boundary. This is a fairly significant restriction; however one will have an infinite number of such manifolds. One obtains these simply by taking the connected sum of V with any closed simply connected 4-manifold. One can readily show that the connected sum of two such manifolds leaves H_k unchanged except for H_2. There are an infinite number of closed simply connected 4-manifolds characterized by their Hirzebruch signature and Euler characteristic. Furthermore the restriction that V is simply connected is not enough to deduce the topology of the boundaries Σ_i even in this simple case. It is well known that all closed 3-manifolds are cobordant to S^3. In fact one can construct a cobordism with trivial fundamental group [20]. Thus, at least by the methods discussed here, the topology of the interior of a $(4+1)$-dimensional ALADS spacetime is constrained but not completely characterized by the topology of the boundary-at-infinity.

We mention in closing that the geodesic methods used to prove topological censorship can be adapted to the Euclidean setting to improve the results of Witten and Yau, cf., [21].

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