Lab 8—Change of Bases

Objective: To become familiar with change-of-basis matrices.

Useful MATLAB Commands:

`inv(M)`	Finds the inverse of the square matrix M, if an inverse exists.
`rref(A)`	Returns reduced row echelon form matrix obtained from A.
`rank(A)`	Returns the rank of A.
`eye(n)`	Creates the nxn identity matrix I_n, where n is a positive integer.

1. Let V be the subspace of R^4 spanned by the basis set *B* comprised of the vectors

$$\mathbf{v}_1 = (2,1,0,1), \quad \mathbf{v}_2 = (1,-1,-1,0), \quad \mathbf{v}_3 = (1,1,2,2).$$

   a) If $\mathbf{x} = (7,0,-4,1)$, show that $\mathbf{x}$ is in V. What are the *B*-coordinates of $\mathbf{x}$? (Hint: You should be able to do both parts of this question at once.)

   b) Find a vector $\mathbf{v}_4$ such that $\{\mathbf{v}_1,\mathbf{v}_2,\mathbf{v}_3,\mathbf{v}_4\}$ is a basis for R^4. [Remark: There are many correct answers.] Check that $\{\mathbf{v}_1,\mathbf{v}_2,\mathbf{v}_3,\mathbf{v}_4\}$ is linearly independent.

Now recall the formula giving the behaviour of a matrix under a change of basis.

If the matrix $[L]_S$ is the standard matrix of a linear mapping L:R^n->R^n and if *B* is another basis set for R^n, then we may write the matrix for L in the B-basis as

$$[L]_B = P^{-1}[L]_S P,$$

where the columns of P are the vectors comprising the basis set *B*.

2. Let L:R^3->R^3 be a linear mapping with standard matrix

$$[L]_S = \begin{bmatrix} 1.4 & 2.1 & -3.5 \\ 3.1 & -4.5 & 1.2 \\ -3 & -2.7 & 3.9 \end{bmatrix}$$

   Let $B = \{(1,1,2),(2,4.3,-2.2),(2.1,-2.5,-0.9)\}$ be a new basis set. What is the matrix $[L]_B$ of this mapping in the new basis set? (Hint: The MATLAB `inv()` command listed above may be useful.)

3. Let $B = \{(1,0,1),(4,1,-2),(-2,2,1)\}$ be a basis of mutually orthogonal vectors on R^3. Let *L* be the linear mapping L:R^3->R^3 described by the *B*-matrix

$$[L]_B = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}.$$

This mapping is a familiar geometrical transformation. Can you identify it? What matrix describes this mapping in the standard basis for R^3?