Lab 7—Bases

Objective: To understand bases.

Useful MATLAB Commands:

rref(A)	Returns reduced row echelon form matrix obtained from A.
rank(A)	Returns the rank of A.
null(A)	Returns one or more columns of numbers. Each column is a basis vector for the nullspace of A.

1. Find the reduced row echelon form (RREF) for the matrix **A** corresponding to the linear mapping $L: \mathbf{R}^5 \to \mathbf{R}^5$, where

$$A = \begin{bmatrix} 2 & 1 & 2 & -1 & -3 \\ -3 & 3 & 2 & 4 & 1 \\ -1 & 4 & 4 & 3 & -2 \\ 3 & 4 & 2 & 1 & -1 \\ 4 & 0 & -2 & -2 & 1 \end{bmatrix}.$$

   Use the RREF to determine a basis for the solution space of the system **Ax** = **0**. From this basis, determine the dimension of the nullspace of the mapping *L* whose matrix is **A**.

   Check your result by using the null(A) command to have MATLAB find a basis for the nullspace of *L* (note this basis may be different from the one you found, but must lead to the same result for the dimension).

   Confirm for this example the nullity theorem (sometimes called the rank-nullity theorem or the dimension theorem), which says that the dimension of the nullspace plus the rank of *L* equals the dimension of its domain.

2. Consider the vector space V = Sp({**a,b,c,d**}) where

$$\mathbf{a} = (2,2,-1,5)$$
$$\mathbf{b} = (-3,6,-1,9)$$
$$\mathbf{c} = (4,-1,2,3)$$
$$\mathbf{d} = (-3,1,3,2)$$

   a) Find a basis set **B** for **V**.

   b) Express the vector $\mathbf{x} = 2\mathbf{a} + \mathbf{b} - 2\mathbf{c} - \mathbf{d}$ as a linear combination of elements of the basis **B**.

3. Consider the matrix $\mathbf{M} = \begin{bmatrix} 4 & 4 & -2 & 5 \\ 9 & 3 & -8 & 2 \\ 5 & -1 & -6 & -3 \\ 13 & -5 & -1 & 3 \\ 8 & -4 & 5 & 6 \end{bmatrix}$.

a) What is the dimension of its rowspace? Examine the reduced row echelon form of this matrix to find a basis for the rowspace of $\mathbf{M}$.

b) What is the dimension of its columnspace? Examine the reduced row echelon form of this matrix to find a basis for the columnspace of $\mathbf{M}$.