Lab 6—Inverses

Objective: To identify invertible matrices and obtain their inverses.

MATLAB Commands:

`C=[1 2 3;4 5 6]` Creates a certain 2x3 matrix and calls it `M`. Use either spaces or commas to separate elements of a row and use semi-colons to separate rows.

`C*D` If `C` and `D` are matrices, the asterix automatically uses matrix multiplication to take the product. Result is stored in variable `ans`, which will be overwritten.

`Z=C*D` As above, but result is stored in variable `Z` for later use. We recommend this form over the one above.

`A=[C,B]` If `C` and `B` are matrices with the same number of rows, use this to append `B` to `C`. In particular, if `C` is the coefficient matrix of a system and `B` represents the "right-hand side" of the system, then this command creates the augmented matrix and stores it as `A`.

`rref(M)` Computes reduced row echelon form of matrix. To store result in variable `A`, type `A=rref(M)` instead.

`rank(A)` Counts the number of leading 1s in the reduced row echelon form of matrix `A`.

`eye(n)` MATLAB's name for the nxn identity matrix; e.g., type `A=eye(3)` to make `A` be the 3x3 identity matrix.

Let's invert the following matrix using MATLAB:

$$M = \begin{bmatrix} 12 & 16 & 20 \\ 2 & 5 & 9 \\ 5 & 6 & 12 \end{bmatrix}$$

`M=[12 16 20;2 5 9;5 6 12]` This time, we will assume the matrix is invertible, but you should test that this is so. How would you do that?

`N=[M,eye(3)]` This creates the augmented matrix for the system MX = I with I the 3x3 identity matrix and `X` the unknown 3x3 matrix which is M^{-1}.

`P=rref(M)` You should know that M^{-1}, if it exists, is comprised of the three right-hand columns of this matrix. Let's now use a MATLAB trick to extract these columns:

`Q=P(:,4:6)` This instructs MATLAB to extract each element in columns 4 through 6 (that's what the `4:6` means) and place these elements in `Q`; the first colon in `Q=(:,4:6)` means "extract every element" of these columns.

We claim that $Q = M^{-1}$. To test this, compute

`Q*P` and

`P*Q` What do you get?

Anyone who has had to compute matrix inverses by hand calculation will certainly be impressed by the speed and ease with which we are able to do this in MATLAB.

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1. For each of the following matrices, compute the rank of M. Carry through the above procedure to obtain the matrix analogous to `Q` above. Compute QM and MQ. Are these products equal to the identity matrix in every case? If not, give an explanation why this is so.

a) $M = \begin{bmatrix} -1 & 3 & 3 \\ 5/4 & 3/2 & 1/2 \\ -1 & -1 & 3/2 \end{bmatrix}$

b) $M = \begin{bmatrix} 3.2 & 2.1 & -1.5 & 1.1 \\ -4.4 & -2.5 & -1.0 & 3.0 \\ -1.3 & 0.4 & 2.2 & -0.9 \\ 0.7 & 0 & 3.0 & 2.2 \end{bmatrix}$

c) $M = \begin{bmatrix} 4 & 2 & -1 & 1 \\ 1 & 3 & -2 & -3 \\ -2 & 1 & 4 & -8 \\ 7 & 5 & 2 & -5 \end{bmatrix}$

2. Consider the inhomogeneous system

$$15x_1 + 6.5x_2 + 7.5x_3 = 10$$
$$-1.5x_1 - 4x_2 - 10x_3 = 3.5$$
$$20x_1 - 35x_2 + 19x_3 = -9$$

Find the inverse matrix of the coefficient matrix of this system. Use this inverse matrix to find a solution directly (by matrix multiplication). Is the solution you found the only solution possible or not, and why do you think so?