Lab 5—Linear Mappings

Objective: To gain familiarity with basic concepts related to linear mapping.

Some MATLAB Commands:

`M=[1 2 3 4;0 1 0 1]`	Creates the matrix $M=\begin{bmatrix}1 & 2 & 3 & 4\\ 0 & 1 & 0 & 1\end{bmatrix}$.
`C=A*B`	Defines the matrix `C` to be the matrix product of `A` times `B`, where `A` and `B` are matrices.
`rref(M)`	Computes reduced row echelon form matrix row-equivalent to `M`.
`null(M)`	Returns a (possibly empty) set of column vectors. These vectors are a spanning set for the solution space of the system $\mathbf{Mx} = \mathbf{0}$.

1. Suppose that $S:\mathrm{R}^2 \to \mathrm{R}^4$ and $T:\mathrm{R}^4 \to \mathrm{R}^2$ are linear mappings with matrices

$$[S]=\begin{bmatrix}1/2 & -1\\ -1/2 & 3/2\\ 1 & 2\\ 5/2 & -1\end{bmatrix},\quad [T]=\begin{bmatrix}3 & 2 & 1 & 3\\ 1 & -1 & -1 & 2\end{bmatrix}$$

   Determine the matrices for $S\circ T$ and $T\circ S$. What is $S(4,-2)$? Find $T(S(-1,2))$.

2. Does $(5,-1,2,-5)$ lie in the span of the vectors $(1,3,-1,-2)$, $(2,3,-2,4)$, and $(1,-2,3,3)$? If so, express it as a linear comination of these vectors. [Hint: Set up the problem as a system of linear equations, convert the system to a matrix, and use `rref()`.] Does $(1,1,3,-5)$ lie in the span of these vectors?

3. Is $\mathbf{y}=(9/2,-5,3,-13/2)$ in the range of the linear mapping L whose matrix is

$$\begin{bmatrix}2 & -1 & -1\\ 1 & 4 & 3\\ -1 & -2 & 1\\ -2 & 2 & 1\end{bmatrix}?$$

   If so, then find $\mathbf{x}$ such that $\mathbf{y} = L(\mathbf{x})$. [Hint: Again, `rref()` and back-substitution may be useful.]