

Lab 3—Linear Systems and Row Reduction

Objective: To learn how to solve linear systems in MATLAB using matrix manipulations (row reduction) and graphical methods.

MATLAB Commands:

[1 2 3;7,-11 2] Create a matrix using square brackets. To separate elements belonging to the same row, use either a space or a comma. Use a semi-colon to start a new row. This example produces the matrix

$$\begin{bmatrix} 1 & 2 & 3 \\ 7 & -11 & 2 \end{bmatrix}$$

and stores it in the variable `ans`. This variable may be overwritten, so let's store it instead in variable `P`.

`P=ans`

`M=[3 -2;4 5;2 1]` Creates a new matrix and stores it in variable `M`.

`M(3,1)=4` Changes M_{31} to 4 (it was previously 2). Use this to correct typing errors when entering matrices. Notice we use parentheses to enclose the indices (which appear as subscripts in your text) which specify which element of the matrix we are working with. Square brackets are used to enclose the matrix elements themselves.

Row Reduction:

`rr(P)` Invokes the `rr` routine installed in our CAB labs (not available elsewhere). You can then perform elementary row operations to reduce `P` to row echelon form. Simply respond when the system prompts you.

1. For each of the following systems, create the augmented coefficient matrix. Use the `rr` routine in MATLAB to obtain a row-equivalent matrix that is in row echelon form. Copy this matrix down and submit it on your answer sheet. Is the system consistent? If no, why do you think so? If yes, solve the system by either back-substitution (by hand), or by complete elimination (to reduced row echelon form), or both, and find all possible solutions. [Notice that the solutions of parts a) and b) belong to \mathbf{R}^3 while the solutions of parts c) and d) belong to \mathbf{R}^2 .]

$$\begin{array}{ll}
\frac{1}{2}x_1 - \frac{1}{4}x_2 = \frac{3}{2} & -x_1 + x_2 = 3 \\
\text{a) } 2x_1 + \frac{1}{2}x_2 = 2 & \text{b) } 3x_1 + 2x_2 = 1 \\
4x_1 - \frac{1}{2}x_2 = 8 & 5x_1 + 4x_2 = 2 \\
\\
3x_1 - 2x_2 + x_3 = 1 & 2x_1 - x_2 - x_3 = 1 \\
\text{c) } x_2 - 2x_3 = 2 & \text{d) } x_1 + x_2 + 2x_3 = -4 \\
x_1 + \frac{1}{2}x_2 - \frac{3}{2}x_3 = 2 & 3x_1 - 3x_2 - 4x_3 = 6
\end{array}$$

Plotting: The following commands will be useful in completing the exercises:

x=linspace(a,b,N) Creates a partition of the interval $[a,b]$ along the x -axis into N evenly spaced x -values from a to b . MATLAB displays a data table containing these values. For example: `linspace(-5,5,100)` partitions the interval from $x = -5$ to $x = 5$ into 100 sub-intervals. In this case, each sub-interval will have width given by

$$\text{width} = (5 - (-5))/100 = 10/100 = 0.1.$$

y=m*x+b MATLAB inputs the x -values defined in the partition above into this formula and computes corresponding y -values. MATLAB displays these y -values in a table. You supply actual numbers for m and b ; for example, to create $y = \frac{3}{2}x - 4$ type `y=(3/2)*x-4`.

plot(x,y) MATLAB now uses the data tables of x - and y -values to construct a plot of x vs. y .

You can plot several graphs on a single set of axes. For example, to plot the graphs of $y = \frac{1}{2}x - 1$ and $y = -x + 3$ on the domain $[0,15]$ on the same set of axes, you can use the following commands

x=linspace(0,15,150) Here we plot 150 points on this interval. The number of points would be important if we were plotting functions more complicated than the straight lines used in this example.

y1=(1/2)*x-1 Call this function `y1`.

y2=-x+3 And call this one `y2`. Obviously we must use a different name for each function.

`plot(x,y1,x,y2)` This plots x vs. y_1 and x vs. y_2 on one set of axes. Notice we must enter x twice, once for each plot.

Similarly, if you have defined y_3 as a function of x , then use

`plot(x,y1,x,y2,x,y3)` to plot all three functions on a single set of axes.

2. (i) Consider the system of equations given in part a) of Question 1. Rewrite each equation so that x_2 is isolated on the left-hand side of the equation. In the first equation, rename x_2 . Let's call it y_1 instead. Instead of x_1 we will simply write x . The equation should now read

$$y_1 = 2x - 6.$$

Rewrite the other equations similarly, but in the second equation rename x_2 and call it y_2 . In the third, call it y_3 . We get

$$y_2 = -4x + 4$$

$$y_3 = 8x - 16$$

Plot these three lines on a single set of axes. Copy down the plot and pass it in. Indicate the solution of the system on the plot.

- (ii) Use a plot to explain geometrically your findings for part b) of Question 1.