Lab 2—Projections

Objective: To practise using projections of vectors to solve geometrical problems.

Recall MATLAB Commands:

Some Arithmetic and Trig:

`x=7`	Assign to `x` the value `7`.
`3*14/(4+2)`	Always use asterix (`*`) for multiplication, slash (`/`) for division.
`3^2`	Use hat (`^`) for exponents.
`sqrt(25)`	Square root function.
`cos(pi)`	Use `pi` for π. The other trig functions are similarly defined.
`acos(1)`	Inverse cosine ($\arccos$). Make inverse trig functions by placing an `a` in front of the command for the trig function.

Some vector commands:

`a=[12 5 -3]`	This creates the vector $(12,5,-3)$ and gives it the name `a`.
`b=(1/3)*[0 10 3]`	Creates vector $(0,10/3,1)$. Notice how arithmetic is done in MATLAB: Slash (`/`) for division and asterix (`*`) for multiplication.
`dot(a,b)`	Takes the dot product of two vectors `a` and `b`.
`cross(a,b)`	Takes cross product `axb`.

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1. Use MATLAB's `dot` instruction to compute the projection $\text{proj}_{\mathbf{a}}\mathbf{b}$ of $\mathbf{b}=(1,-4,2)$ along the direction of $\mathbf{a}=(-3,1,1)$. Find $\text{perp}_{\mathbf{a}}\mathbf{b}$. Finally, check that $\text{proj}_{\mathbf{a}}\mathbf{b} + \text{perp}_{\mathbf{a}}\mathbf{b} = \mathbf{b}$ and that $\text{proj}_{\mathbf{a}}(\text{proj}_{\mathbf{a}}\mathbf{b}) = \text{proj}_{\mathbf{a}}\mathbf{b}$.

2. Find the projection of the vector $\mathbf{b}=(4,1,-3)$

   i) in the direction perpendicular to the plane $2x+3y+z=4$.

   ii) parallel to the plane $2x+3y+z=4$.

3. In this problem, we will use the cross product and projection to find the distance between the lines $\mathbf{x}(t)=(1,1,2)+t(2,-3,2)$ and $\mathbf{x}(s)=(0,1,-1)+s(1,1,3)$.

   (i) Find a vector $\mathbf{n}$ that is perpendicular to the tangents to both these lines.

(ii) Choose any two points, one on each given line, and find the vector **v** joining your chosen points.

(iii) Find and interpret the component of **v** along the direction of **n**.

(iv) Can you give a reason why the two given lines are not parallel? Using only the result of part (iii), can you give a convincing reason why the two given lines do not intersect? If so, then one can conclude these lines are skew lines.