Lab 16—MATLAB's Symbolic Toolbox

Objective: To become familiar with MATLAB's facilities for handling symbolic expressions.

MATLAB Commands:

`x=sym('x')`	Defines `x` to be a symbolic object which can then be treated as a mathematical variable.
`syms('a','b','c','d')`	Creates several symbolic objects at once (`syms('a')` is the same as `a=sym('a')`).
`syms a b c d`	Same as above.
`y=sym('1/2')`	Creates symbolic object `y` and then gives it the definite value 1/2, so `y` is treated as a mathematical constant.
`M=[a b;c d]`	Creates symbolic matrix, if `a`, b, `c`, and `d` are symbolic objects.
`ezplot(y,[a b])`	Plots symbolic expression `y` (given as a function of symbolic object `x`) on the interval from `a` to `b`.
`factor(y)`	Factors symbolic expression `y` (given as a function of symbolic object `x`).
`S=solve(eqn1,eqn2,...)`	Solves a system of equations in symbolic variables `x1`, `x2`, `...` and stores the solution in `S`.
`S.x1`	Returns the value that `x1` takes in the solution `S`.

Unlike other mathematical software packages such asMaple, basic MATLAB deals only with numbers and not with symbols. To work with symbols, MATLAB has an extension called the *symbolic toolbox*, which is actually a part of the Maple software package appended to MATLAB. The syntax for using the symbolic toolbox is cumbersome, but should improve in future versions. In this lab, we will become familiar with just a small portion of the toolbox's capabilities.

1. Plot the polynomial $y = x^3 - 3x + 2$. Factor y and find all values of x such that $y = 0$. Remember when defining `y` to first define `x` as symbolic. Also remember that every multiplication must be represented by an asterix (`*`) and that exponents are preceded by the hat (`^`) symbol.

To solve an equation or a system of equations in the symbolic toolbox, rewrite each equation if necessary so that zero appears as the right-hand side. Give the left-hand side of each equation a name. For single equations, we apply the

`solve()` command directly to this name. For systems, we first apply the `solve()` command and then ask for the value of each unknown variable in turn.

For example, to solve $x^2 - 3x + 1 = 5$, we can rewrite it as $x^2 - 3x - 4 = 0$. Then:

```
syms x
eqn=x^2-3*x-4
solve(eqn)
```

Try it yourself. Notice you get two roots, of course.

Next, let's try the inhomogeneous system

$$x_1 + 2x_2 = 3$$
$$4x_1 + x_2 = -2$$

Now the syntax is more complicated:

```
eqn1=x1+2*x2-3
eqn2=4*x1+x2+2
S=solve(eqn1,eqn2)
```

Notice the `S=` (you can use any symbol you may prefer instead of `S`, if you wish).

```
S.x1
```

This returns to value of `x1` in the solution.

```
S.x2
```

This returns to value of `x2` in the solution.

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2. Use the symbolic toolbox to solve the system

$$x_1 + 2x_2 + 2x_3 = 0$$
$$2x_1 + 3x_2 + 4x_3 = 1$$
$$-x_1 + x_2 + x_3 = -6$$

Finally, we will use the symbolic toolbox by using it to find the solutions of the equation

$$\det(A - xI) = 0$$

where A is a square matrix, I is the identity matrix, and x is a variable. You may already know that the solutions x of this equation are called *eigenvalues* of A.

For example, to find the eigenvalues of $A = \begin{bmatrix} 1 & 0 & -1 \\ 1 & 2 & 1 \\ -1 & 1 & 0 \end{bmatrix}$, we might use the commands

```
A=sym('1 0 -1;1 2 1;-1 1 0')
```

Notice we declare `A` to be symbolic even though it contains no symbols. We do this so that we can subtract from it the following matrix, which does contain symbols.

```
B=sym('x 0 0;0 x 0;0 0 x')
```

Thus `B=xI`.

`p=determ(symsub(A,B))` Then `p=det(A-xI)`. Notice that `p` is a polynomial, the so-called *characteristic polynomial* of `A`.

`s=solve(p)` We store the roots of the polynomial `p` in the computer's memory in a variable called `s`.

If this result is not in numeric form, then we apply one last command:

`numeric(s)` This is only necessary if the quantities stored in `s` are not already in decimal form.

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3. Use the symbolic toolbox commands listed above to compute all eigenvalues of the matrix

$$A = \begin{bmatrix} 4 & 8 & 5 & 3 \\ -1/2 & 0 & -1/2 & 0 \\ -5/2 & -7 & -7/2 & -3 \\ 3/2 & 3 & 3/2 & 2 \end{bmatrix}$$