Lab 12—Diagonalization

Objective: To practise diagonalization of matrices over the real numbers.

MATLAB Commands:

`eig(A)`	Returns the eigenvalues of square matrix A.
`det(A)`	Computes the determinant of square matrix A.
`inv(A)`	Gives the inverse of square matrix A.
`eye(n)`	This is the nxn identity matrix—handy for eigenvalue problems.

Note: In this lab, we use different notation than your text. We will use **v** for eigenvectors (your text uses **x**) and x for eigenvalues (your text uses λ).

1. a) Use MATLAB's `eig()` command to find the eigenvalues of $M = \begin{bmatrix} -3 & -5 & -7 \\ -2 & 1 & 0 \\ 1 & 5 & 5 \end{bmatrix}$.

   b) For each eigenvalue x found above, find the corresponding eigenvector **v** of M by using row reduction to solve $(M - xI)\mathbf{v} = \mathbf{0}$.

   c) Construct a matrix P whose columns are the eigenvectors of M. Compute the product $D = P^{-1}MP$ and confirm that D is diagonal. Compute the determinants of D and M and confirm that they are equal.

   d) Construct another matrix Q whose columns are also the eigenvectors of M but this time place them in a different order than in P (so perhaps the first column of P is the second column of Q and so forth). Again compute $P^{-1}MP$. What has changed?

2. Find a basis for R^4 comprised of eigenvectors of the matrix $A = \begin{bmatrix} 1 & 1 & -1 \\ 0 & 3 & -2 \\ 0 & 0 & 1 \end{bmatrix}$.

   Using it, construct a matrix P that diagonalizes A. Compute $P^{-1}AP$.

3. <u>The square root of a matrix:</u> By any convenient method diagonalize the matrix $M := \begin{bmatrix} 46 & 42 \\ -21 & -17 \end{bmatrix}$. Construct the matrix P whose columns are the eigenvectors of M. As well, construct a diagonal matrix Q whose diagonal elements are the square roots of the eigenvalues of M (the off-diagonal

elements are of course zero). Lastly, find the matrix $B = PQP^{-1}$. Verify that $B^2 = M$. In this sense, B is a "matrix square root" of M.