Lab 11—Determinants and Eigenvalues

New MATLAB Commands:

`det(A);`	Takes determinant of matrix `A`.
`poly(A)`	Returns the coefficients of the characteristic polynomial of A. For example, the matrix $A=\begin{bmatrix}0 & 3\\ 1 & 2\end{bmatrix}$ has characteristic polynomial $\lambda^2-2\lambda-3$, so the command `poly(A)` returns: `ans =  1  -2  -3`
`roots(p)`	If `p` is a polynomial (for example, if `p=poly(A)`), then this command returns the roots of `p` (recall the roots are the values of the independent variable that make `p=0`).

Recall:

`rr(A);`	Permits you to carry out elementary row operations on `A`.

1. a) Use elementary row operations to reduce $M=\begin{bmatrix}2 & 1 & 2 & 3\\ -3 & 1 & 1 & 2\\ 2 & 3 & 1 & 2\\ -2 & -1 & 3 & 2\end{bmatrix}$ to upper triangular form. [You can do this by hand or using MATLAB. If you use MATLAB, use the `rr()` routine, not `rref()`, since in order to answer part (b) you will want to know specific information concerning the row operations performed.]

   b) Use this upper triangular form and your knowledge of the elementary row operations used to obtain it to compute the determinant of M. Check using MATLAB's `det()` command.

2. Use Cramer's Rule to solve the following system:

$$\begin{aligned} x_1+2x_2+3x_3+4x_4&=0\\ x_1-x_2\ \ +x_3\ \ -x_4&=1\\ -x_1+2x_2\ \ -3x_3\qquad&=1\\ 2x_1+2x_2\ \ -2x_3+x_4&=2 \end{aligned}$$

   [Hint: Do the determinants using MATLAB's `det()` command and then plug them into Cramer's Rule by hand. To save typing, first create the coefficient matrix of this system and call it `A`. Make a copy `C` to work on by typing `C=A` (don't destroy `A`; we'll need to recycle it). Create a column vector whose entries are the right-hand side of these equations and call it `b`. To replace, say, the second column of `C` by `b`, type `C(:,2)=b`.]

3. Compute by hand the characteristic polynomial of the matrix

$$A = \begin{bmatrix} 5 & 3 & 6 \\ 2 & 6 & 6 \\ 2 & 3 & 9 \end{bmatrix}$$

Check using MATLAB's `poly()` command (remember, this command returns only the coefficients of the polynomial—the variable itself will be missing). Use the `roots()` command to find all eigenvalues. Use you knowledge of the eigenvalues to write the characteristic polynomial as a product of three factors.