

## Math 201 A1—Maple Lab 7

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**Objective:** To gain familiarity with Laplace transforms, including the Laplace transforms of step functions and related functions.

### Maple Commands:

**with(inttrans):** Loads the integral transforms package containing laplace transform commands.

**laplace(expr, t, s);** Applies laplace transform to `expr` with variable `t`. Expresses result as a function of variable `s`.

**invlaplace(expr, s, t);** Inverse transform.

**Heaviside(t);** The Heaviside step function.

**f:=(x,y)->expr;** Maple notation for a function that accepts `x` and `y` as input and returns as output the expression `expr`.

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Laplace transforms are most often used to solve differential equations with discontinuous or, more generally, non-smooth inhomogeneous terms. We will address this issue in future labs. In this lab, we will simply endeavor to become accustomed to Laplace transforms. We will compute some by hand and check them with Maple, and will introduce the idea of the Heaviside step function.

Let's start by having Maple compute the Laplace transform of  $t^n$  for  $n$  a positive integer. We first load the *integral transforms* package:

```
with(inttrans):
```

If you simply type

```
laplace(t^n, t, s);
```

then Maple will complain that it needs to know something about  $n$ . We can tell it what it needs to know:

```
assume(n>0);
```

Maple will now replace  $n$  by  $n\sim$  in all output. The tilde symbol ( $\sim$ ) indicates that  $n$  is now a *constrained* variable. (To release the constraint, you would type `n := 'n'`.)

```
laplace(t^n, t, s);
```

One further matter: Maple returns an answer that involves the *Gamma function*, which is discussed in the text. All we need to know about it here is that  $\Gamma(n+1) = n!$  for  $n$  a positive integer, where the exclamation point denotes the factorial.

Exercise 1: By hand calculations, compute the Laplace transforms of  $t$  and  $t^2$  (hint: knowledge of the transform of  $t$  will help you compute the transform of  $t^2$ ). Compare your answers to what you get directly from Maple's `laplace()` command.

In the course, you will soon encounter the useful notion of a step function, also called a Heaviside step function in honour of a mathematician of the last century. This function has value zero if its argument is negative and one if its argument is positive.

$$H(t-a) = \begin{cases} 1 & \text{if } t > a \\ 0 & \text{if } t < a \end{cases}$$

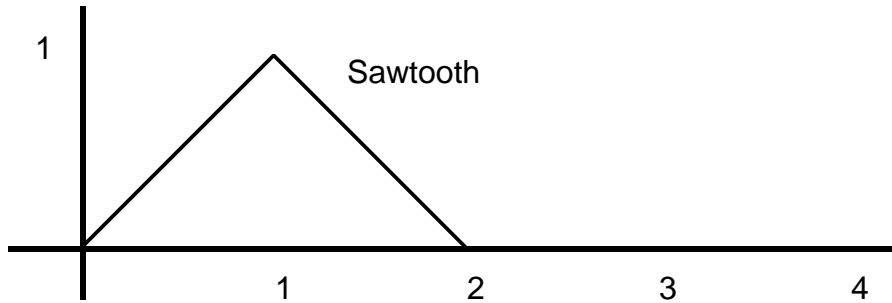
Maple calls this function `Heaviside()`. We can use it to make other useful functions, such as the square pulse of width 1, height 1, and initial point  $t_0$ . This function has value 1 if  $t$  lies in the interval  $(t_0, t_0+1)$ , and is zero outside this region. To construct it, multiply together two step functions:

$$\text{sqPulse}(t_0, t) = H(t-t_0)H(t_0+1-t)$$

```
sqPulse := (t0, t) -> Heaviside(t-t0)*Heaviside(t0+1-t);
plot(sqPulse(2, t), t=-2..5);
```

Exercise 2: Compute by hand the Laplace transform of the square pulse of height 1 that is turned on at  $t=t_0=0$  and turned off at  $t=1$ . Check your result using Maple.

Exercise 3: Using Heaviside step functions, can you write a mathematical formula to describe the single sawtooth, consisting of a line from  $(0,0)$  to  $(1,1)$  and a line from  $(1,1)$  to  $(2,0)$  (the function is zero elsewhere)? Have Maple compute the Laplace transform of your formula. [Hint: Apply the `simplify(%)` command to have Maple simplify its answer: recall the percentage sign `%` means "the last answer" in Maple.]



Exercise 4 (Optional): A *square wave* consists of a periodic sequence of square pulses of height 1 and width 1, repeating with a period of 2, so that, starting at  $t=0$ , the pulse turns on until  $t=1$ , then turns off until  $t=2$ , then turns on until  $t=3$ , ... Can you write a compact mathematical formula for the square wave (using sigma notation to denote infinite sums)?

