Math 201 A1—Maple Lab 3

<u>Objectives:</u> To examine the validity of the small angle approximation in the study of pendulum motion, using Maple's numeric differential equation solver.

<u>Maple Commands:</u>

`diff(y(x),x$2);` Maple notation for the second derivative $y''(x)$.

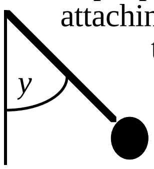
`dsolve(deq,y(x));` Finds general solution to the differential equation `deq`.

`dsolve({deq,y(a)=y0,D(y)(a)=y1},y(x));` Solves initial value problem with differential equation `deq` and initial conditions $y(a)=y_0$, $y'(a)=y_1$.

`plot({expr1,expr2,...},x=a..b);` To plot more than one expression on a single set of axes, enclose the list of expressions in curly braces.

`sin(Pi*x);` Maple notation for $\sin(\pi x)$. Notice the Maple name `Pi` for the constant π must begin with a capital letter.

`f:x->expr;` Maple function.



A simple pendulum is constructed by attaching a mass to one end of an arm of length l and attaching the other end of the arm to a pivot. If the pendulum swings freely, the angle that the arm makes to the vertical is given by the differential equation $\frac{d^2\theta}{dt^2}+\frac{g}{l}\sin\theta=0$ where g is the acceleration due to gravity and t denotes the time. We will make the simplifying assumption that the length of the pendulum arm is chosen so that $l=g$. Also, we will use y instead of θ to denote the angle. Then the pendulum equation is

$$y''(t)+\sin(y(t))=0 \qquad (1)$$

For small angles, $\sin(y)\approx y$ and so we expect the pendulum in this case to obey

$$y''(t)+y(t)=0 \qquad (2)$$

<u>Exercise 1:</u> Write the general solution of (2) and the particular solution that obeys $y(0)=c$, $y'(0)=0$. This corresponds to a pendulum which is brought to rest at an initial angle c and gently released. The time between two successive passes of the pendulum through angle $y=0$, swinging in the same direction each time, is called the period of the pendulum. What is the period of this pendulum? Does it depend on c?

<u>Numeric Solutions, and how to plot them:</u>

If c is small, then the solution to equation (2) that we just obtained should be reliable. However, if c is large, we expect to need equation (1). We have no technique for solving this equation. Moreover, even Maple can't fully solve it—it can write the solution, in implicit form, as an integral but it cannot do this integral.

```
deq1:=diff(y(t),t$2)+sin(y(t))=0;
dsolve(deq1,y(t));
```

In such cases, we ask Maple to use a numerical method to do these integrals. Maple will only do so if we fully specify numerical values for the initial data. Here's an example corresponding to gentle release of a pendulum from an angle of 0.1 radians with respect to the vertical:

```
soln:=dsolve({deq1,y(0)=0.1,D(y)(0)=0},y(t),numeric);
```

Notice the important word `numeric`. Don't expect an explicit answer: Maple has computed a table of numeric values for $y(t)$, which we now have to display. We can display individual values, but let's display a plot. The tool needed for this belongs to the `plots` package.

`with(plots):` We load in the `plots` package.

`odeplot(soln,x=-4*Pi..4*Pi);` The `odeplot()` command is a version of `plot()` that can handle numeric output from `dsolve()`.

Last lab, we learned how to plot many different solutions on one graph by varying the initial data. We'd like to do that here, but we have a problem. How can we vary the initial data since we had to choose specific data before calling `dsolve( ,numeric)`?

To solve this problem, we create a Maple function. The input to the function is the initial data, and the output is the numeric solution. Actually, we use two functions together; the second one turns the numeric solution into a plot. We vary the input and plot the results.

```
curv:=c->dsolve({deq1,y(0)=c,D(y)(0)=0},y(t),numeric);
graph:=c->odeplot(curv(c),[t,y(t)],0..5);
```

Here the plot domain is $t \in [0,5]$. You may want to use a larger domain for the exercises.

```
display(graph(0.1),graph(0.2),graph(0.3),graph(0.4));
```

An equivalent way to enter this last command is to createe a list using a sequence. The syntax is not too hard to understand:

```
myList:={seq(graph(0.1*i),i=1..4)}:
display(myList);
```

Exercise 2: For initial data $y(0) = 1$, use a graph to estimate the period of the pendulum. How does it compare to the period 2π obtained from the approximate equation `(2)`? Does the period appear constant over time? Does the amplitude (that is, the maximum value of y) vary with time? On one set of axes, plot several solutions, corresponding to a range of initial angles between 0.25 and 1.25 radians, say in increments of 0.25. Pass in a copy of the plot with each curve labelled by the initial data used to generate it. Describe the dependence of the period on the initial angle from which the pendulum is released.

Exercise 3: Define a Maple function that will allow you to plot several solutions of equation `(1)`, but this time the initial data should be of the form $y(0) = 0$, $y'(0) = v_0$. In other words, the pendulum will initially hang vertically but will be given a push, acquiring an initial velocity v_0. On a single set of axes, plot several solutions for different initial velocities; pass in a labelled copy of this graph. How does the period vary with the initial velocity? Based on your results from the previous exercise, what sort of variation should you have expected?

Solve the approximate equation `(2)` with the initial data $y(0) = 0$, $y'(0) = v_0$. Does this equation predict the period will vary with the initial velocity?