Math 201 A1—Maple Lab 2

Objectives: We will gain further experience with certain features of Maple that are useful for solving differential equations by considering questions of uniqueness and stability of solutions.

Maple Commands:

`dsolve(deq,y(x));` Finds general solution to the differential equation `deq`.

`dsolve({deq,y(a)=y0},y(x));` Solves initial value problem with differential equation `deq` and boundary (or initial) condition $y(a) = y_0$.

`infolevel[dsolve]:=2;` Provides more information on the workings of `dsolve()`. To turn it off, set `infolevel[dsolve]:=1;`

`plot({expr1,expr2,...},x=a..b);` To plot more than one expression on a single set of axes, enclose the list of expressions in curly braces.

`exp(t);` Maple notation for the exponential function e^t.

`diff(y(x),x);` Maple notation for the first derivative $y'(x)$.

Let's start by practising use of the `dsolve()` command:

Exercise 1: Decide whether each of the following equations is separable, linear, Bernoulli, or some other type. An equation can be more than one of these types; if so, give all the possibilities. Set `infolevel[dsolve]` to `2` and submit each equation to `dsolve()`. In each case, what type does Maple decide the equation is? In each case, give the general solution and the particular solution that satisfies the initial condition $y(0) = 1$:

a) $\frac{dy}{dx} = (x + y)^2$

b) $\frac{dy}{dt} = t^3 y + y$

For first-order initial value problems, if the differential equation has well-behaved coefficients, so-called existence and uniqueness theorems tell us that there always is exactly one solution. But Maple doesn't always know this. This problem sometimes arises when Maple takes square roots. Maple often returns two solutions, one for the positive square root and one for the negative.

Exercise 2: Consider:

$$\frac{dy}{dx} = \frac{y + x}{y - x} \quad , \quad y(0) = 1.$$

Does your Maple system claim this problem has more than one solution? If so, check to see if all solutions obey the initial condition or not. What type of differential equation is this?

In many situations, it is important to know how the solution of an initial value problem changes in response to a change in the initial condition. To understand this issue, we would like to be able to plot a whole family of solutions; that is, we would like to plot several different solutions, each corresponding to different initial data, so that we can compare them.

There is a Maple trick for doing this (we could also do it using `DEplot()`, but there are reasons for exploring other methods). Work through the following example to learn the trick. Let's solve the linear equation

$$\frac{dy}{dt} - 2y = e^t$$

The first part of the trick is to solve the equation and to specify initial data of the form $y(0) = c$. We can then vary the value of c and see how the solution responds.

```
sol:=dsolve({diff(y(t),t)-2*y(t)=exp(t),y(0)=c},y(t));
```

Now we can't plot this because it isn't a proper Maple expression or function. The trick is to extract the rhs (right-hand side) of the equation using Maple's `rhs()` command.

```
expr:=rhs(sol);
```

This is a Maple expression, but we still can't plot it properly, because it depends on two "variables" t and c instead of just one. We first have to give a value to c. But what we want to do is give a sequence of values instead of just one, so we can compare different solutions belonging to the different initial values $y(0) = c$. This is conveniently done using Maple's *sequence operator* `$`:

```
plot({expr$c=-3..3},t=-1..1);
```

This instructs Maple to let c take values -3, -2, -1, 0, 1, 2, and 3. Notice the curly braces. They should not be a surprise; you probably already know that when Maple plots more than one function on a single set of axes, it expects you to enclose the list of functions in curly braces. The only new thing here is convenient abbreviated notation for the list.

Notice that the different solutions diverge away from each other as t increases. You can confirm this behaviour by choosing a larger domain. Try

```
plot({expr$c=-3..3},t=-4..4);
```

Now you can see that just a small difference in initial conditions at $t = 0$ leads to a very big difference by the time we reach $t = 4$. The solutions become very sensitive to initial conditions as t increases. In this case, we say the differential equation is *unstable* as $t \to \infty$. This particular equation is *stable* as $t \to -\infty$, since the solutions are clearly *insensitive* to the initial data there (they all approach zero).

__Exercise 3:__ For several values of k between –5 and 5, have Maple solve the initial value problem

$$\frac{dy}{dt} + ky = e^t \quad , \quad y(0) = c \, .$$

For each case, use a plot of several solutions corresponding to different initial values c to determine whether the problem is stable as $t \to \infty$.

Now use hand calculations to solve this initial value problem (you may ignore the case $k = -1$, which requires special treatment). By considering the limit of your solution as $t \to \infty$, can you confirm the stability behaviour you just discovered by looking at the plots?

__Exercise 4 (Optional):__ Solve the initial value problem in Exercise 2 using hand calculations.