Math 201A1—Maple Lab 1

Reminder: Below you will find some *Exercises*. For these, copy the answers from the screen onto paper and pass in the paper with your name on it.

Objectives: In this lab, we will familiarize ourselves with Maple's basic tools for solving and analyzing differential equations (DEs).

Recall Maple Commands:

`int(expr,x);` Indefinite integral of Maple expression `expr`.

`int(expr,x=a..b);` Definite integral of Maple expression `expr` on domain $[a,b]$.

`plot(expr,x=a..b);` Plots Maple expression `expr` on domain $[a,b]$.

`plot(expr,x=a..b,y=c..d);` Plots Maple expression `expr` on domain $[a,b]$ and range $y=[c,d]$.

To begin, follow through the commands below in order to solve the differential equation

$$\frac{dy}{dx}=\frac{xy}{x^2+1}. \qquad (1)$$

`deq:= diff(y(x),x)=x*y(x)/(x^2+1);` Creates the differential equation and uses the assignment operator `:=` to assign it the name `deq`. Notice we say `y(x)`, not just `y` here.

`dsolve(deq,y(x));` Solves the differential equation `deq` for `y(x)`. Maple uses `_C1` to denote the arbitrary constant of integration.

`soln:=dsolve({deq,y(0)=5},y(x));` Solves `deq` subject to the boundary condition `y(0)=5`. Notice the curly braces and notice we've given the solution a name, `soln`.

The above are the basic tools for solving DEs in Maple. We will introduce more sophisticated versions of these commands later. Let's now plot the solution. The name `soln` applies to a whole equation `y(x)=...`, but we want to plot an expression (the right-hand side of this equation), so we first extract it:

```
expr:=rhs(soln);
plot(expr,x=-2..2,y=-10..10);
```

Next we turn our attention to the `DEtools` package and some of its features.

`with(DEtools):` Loads the `DEtools` package, containing `DEplot()`.

```
DEplot(deq,y(x),x=-2..2,y=-10..10);
```

This last command plots the *direction field* of the differential equation. In other words, it plots small line segments whose slopes are given by the right-hand side of equation (1). Graphically, finding solutions of (1) amounts to joining up these line segments to create *solution curves* (also called *flow lines* or *integral curves*). Let's use this method to create the graph of the solution that obeys the initial condition `y(0)=5`:

`DEplot(deq,y(x),x=-2..2,{[0,5]},y=-10,,10);` Notice how we specify the initial data `y(0)=5`. The curve should look familiar.

Finally, let's plot several different members of a whole *family* of solution curves, each satisfying different initial data:

```
DEplot(deq,y(x),  x=-2..2,{[0,5],[0,2],[0,-2],[0,-5]},
y=-10..10);
```

Exercise 1: Solve each of the following DEs and initial value problems by hand (you can have help doing integrals from Maple's `int()` command if you wish). Check your solutions by also solving them with Maple.

a) $\frac{dy}{dx} = \frac{x^3}{y}$

b) $y' - y^2 - 1 = 0, \quad y(0) = 1$

Exercise 2: For each of the following DEs, have Maple plot the direction fields on the region $x \in [-2,2]$, $y \in [-2,2]$. By inspecting the direction fields, try to guess how the solution curves will look for different initial data. Can you suggest how solutions might behave as $x \to \infty$? as $x \to -\infty$? Does it depend on the initial data? Choose four different initial data and ask Maple to plot the solution curves for these data. Copy and submit the plot (you should sketch the curves, but not the direction field). Try to ensure as far as possible that you've chosen your initial data so that the resulting solution curves represent the full variety of different shapes that solution curves of these DEs can exhibit.

a) $y' = y^2 - x^2$

b) $\frac{dy}{dx} + 2xy = 2$

Exercise 3: By any convenient method, find the general solution of the separable equation

$$\frac{dy}{dt} = y^2 - y. \qquad (1)$$

Consider initial data of the form $y(0) = a$. First choose some specific values of $a < 0$, determine the corresponding functions $y(t)$ that satisfy these initial data, and have Maple plot them (on a single set of axes). Repeat, choosing this time new a-values $0 < a < 1$. Lastly, repeat once more with a-values greater than 1. (You do not have to pass in copies of these plots.)

Do any of these solution curves cross the line $y = 0$? Do any cross the line $y = 1$? From equation (1), can you give a mathematically sound argument supporting your observations regarding whether solution curves can cross these lines?