Calculus Lab 8—Graphs of Common Functions

Objective: To become familiar with some general characteristics of the graphs of polynomials and periodic functions and to study transformations acting on these graphs.

Maple Commands:

`plot(expr,x=a..b);` Plots expression `expr` on the domain $x \in [a,b]$.

`plot(expr,x=a..b,y=c..d);` Plots expression `expr`; restricts both domain (x-values) and range (y-values).

`plot(expr,x=a..b,y=c..d,discont=true);` Improves appearance of graphs of functions that have a discontinuity.

`plot({expr1,expr2},x=a..b);` Plots both `expr1` and `expr2` on one set of axes.

`with(plots):` Loads the full `plots` package, which contains Maple code for the next command:

`animate(expr,x=a..b,k=c..d,frames=n);` Creates an animated graph of `expr`. The parameter k appearing in `expr` increases from $k = c$ to $k = d$ in successive frames of the animation. Here n is the total number of frames in the animation.

In this lab, you will answer a series of questions concerning the shapes of various graphs. As a general strategy, try to guess the answers first, then view the graph and see if you were right or not. The questions concerning shape are meant to illustrate important features that graphs of similar functions have in common. There is a reason for each such feature try to see in each case what that reason is.

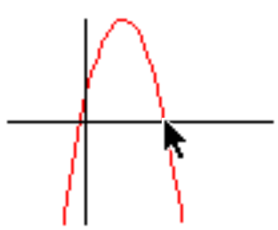
Some of the following questions ask you to sketch graphs. You may use Maple and then copy the results from the screen to your answer sheet. It is not necessary that your copies be overly accurate, *but they must reflect the essential features of the graph*. Pay particular attention to the location of *local maximum* and *local minimum* points and points where the graph intersects one of the axes.

A *local maximum* is a point on the graph that is higher than any nearby points; a *local minimum* is lower than any nearby points.

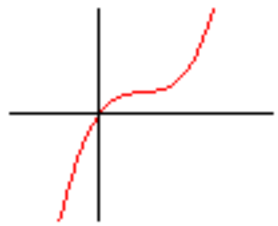
Exercises:

Q1. Here are three graphs. For each graph, list a few general features of the graph and write down a function whose graph shares these features. Using

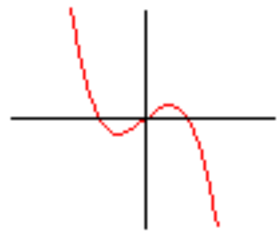
Maple's `plot()` command to graph your function. The graph you obtain should resemble the given graph if not, try changing your function.



Graph A



Graph B



Graph C

Q2. Sketch graphs representing cubic and quartic polynomials (a quartic polynomial is one in which x^4 is the highest power appearing). Your graphs should be "generic" in the sense that they should have all the features you would expect to be present in a completely general polynomial of the required type. [Hint: If unsure, ask Maple to plot a few randomly chosen cubic and quartic polynomials.] Describe in words the general features of these graphs. In particular, note how many local maximum and local minimum points you expect to find.

Q3. Use Maple to sketch the graph of $f(x)=\dfrac{x^2+1}{x^2-4}$. Because this function is discontinuous somewhere (where?), your graph will probably look rather bad unless you restrict the range of the y-axis and use `discont=true` in the `plot()` command (see top of first page). What are the essential features of this graph? When x takes very large values, the graph will approach very closely a straight line, called an *asymptote*. What is this line (*i.e.* give the equation of this line—it will probably help if you have Maple make a new plot using large x-values in the domain)? What asymptote does the graph approach when x becomes very negative?

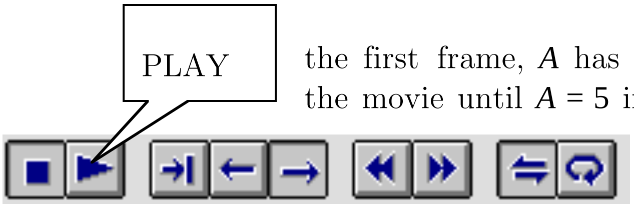
The following questions pertain to *transformations* of graphs, such as *translations* and *scalings*. We will study these using the function

$$f(x)=A\sin(x+c) \tag{1}$$

where A and c are constants. We wish to study what happens as we choose different values for these constants. To do this, try the following animation—be sure to load the full `plots` package first:

```
with(plots):
animate(A*sin(x),x=-4*Pi..4*Pi,A=-5..5,frames=64);
```

and click the `play` button when it appears in the toolbar. You will see a movie whose frames are graphs of $f(x)=A\sin x$. In



the first frame, A has value A = -5, which increases during the movie until $A = 5$ in the final frame.

Q4. Returning to equation (1), fix $A = 1$ and make a movie in which c varies from 0 to 2π. As c increases, in which direction does the graph move, left or right?

Q5. A certain function is of the form given by equation (1). Its graph crosses the x-axis at $x = \pi/4$. The function is positive when $x = 0$. The y-value of the highest point on the graph is $y = 3$. Write down this function (in other words, find A and c). Use Maple to plot it.

Q6. Using Maple movies, describe how the shapes of the graphs of the following functions change when the values of constants are changed. Describe also those aspects of the shapes that do not change.

(a) $f(x) = ax^2 + bx + c$ when c changes. [Choose some particular values for a and b and let c vary from frame to frame in the animation. Repeat once or twice with different choices of a and b to make sure your choices don't strongly affect your observations.]

(b) $f(x) = ax^2 + bx + c$ when a changes. [You'll have to choose some specific values for b and c this time.] What happens when a changes sign?

(c) $f(x) = \cos(kx) + B$ when B changes. Using this result and your answer to part (a), can you make a general remark about what happens to the graph of any function of the form $f(x) = g(x) + B$ when B changes?

(d) $f(x) = \cos(kx) + B$ when k changes.